An Approximation Algorithm for Path Computation and Function Placement in SDNs

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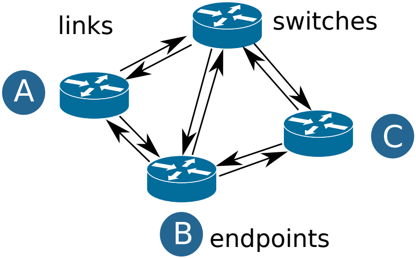
Joint work with *Guy Even* and *Stefan Schmid*

Introduction

Opportunities: Software-Defined Networking (SDN) and Network Functions Virtualization (NFV) [Kreutz et al., 2015]



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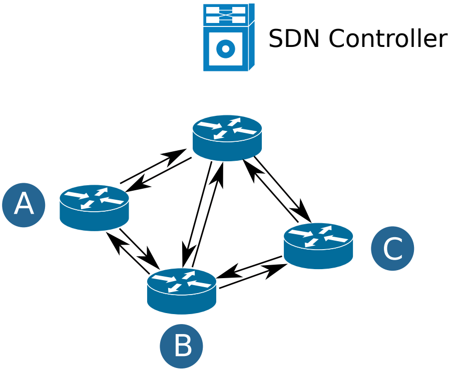

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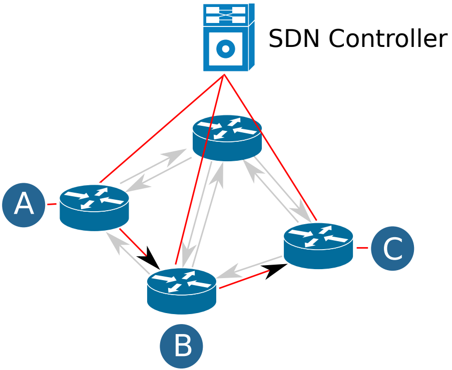

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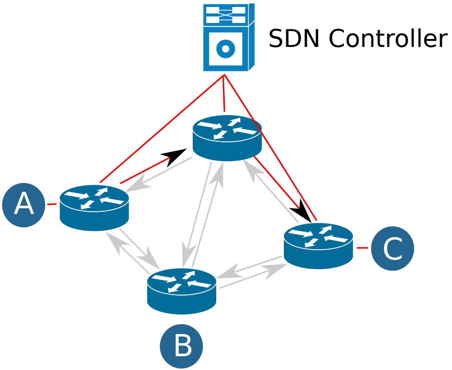

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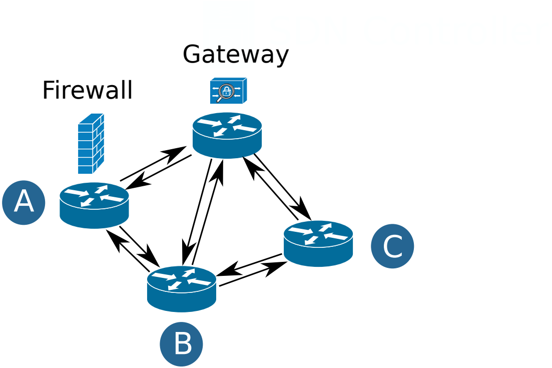

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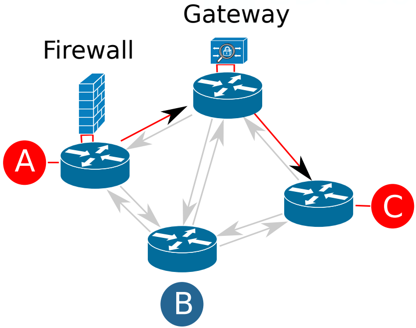

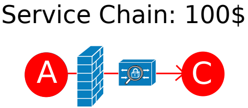

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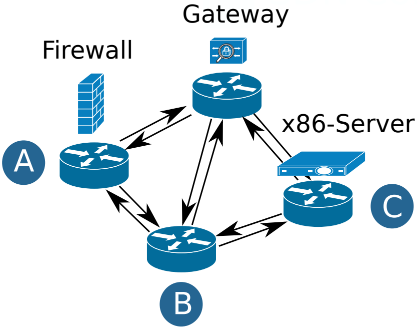

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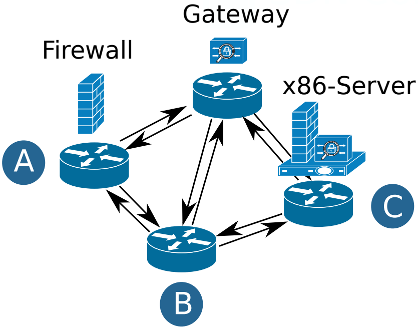

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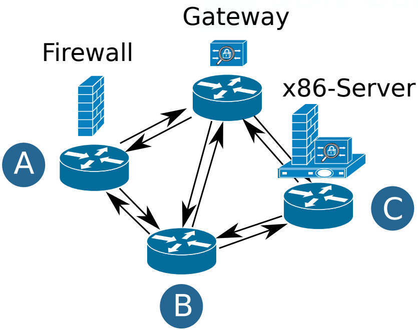

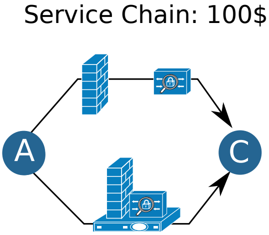

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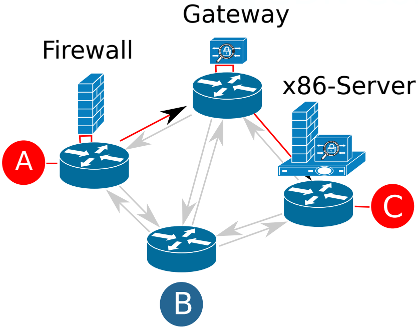

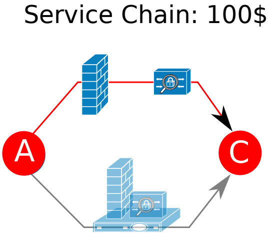

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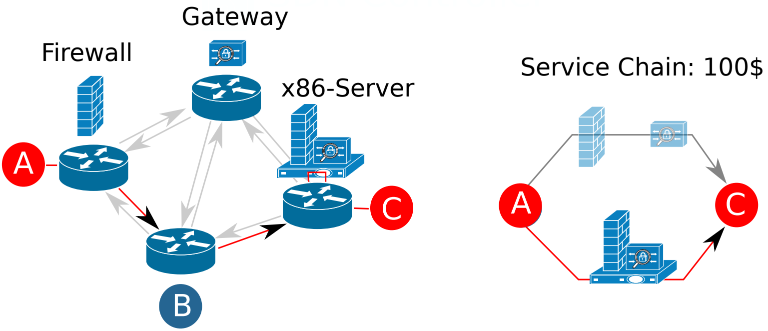
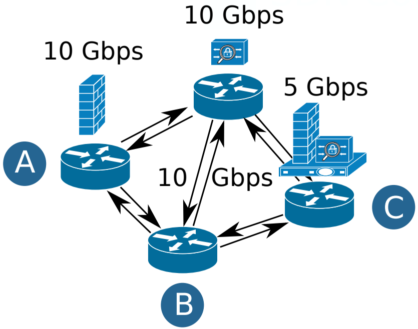
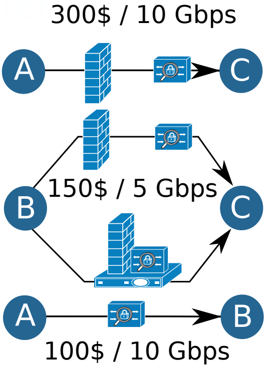


Path Computation and Function Placement Problem (PCFP)





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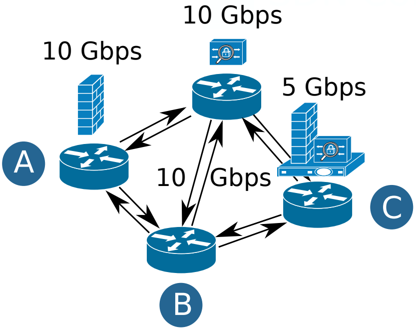

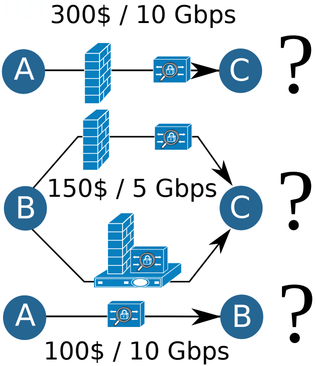

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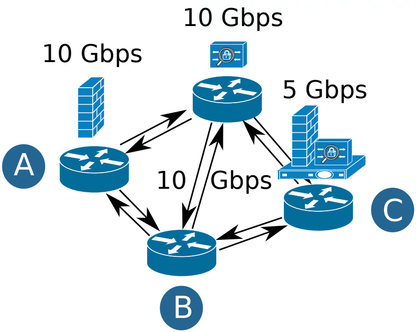

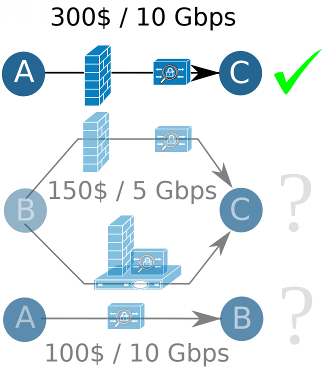

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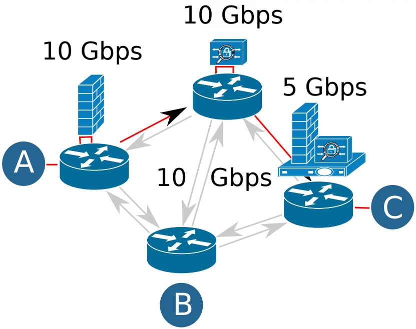

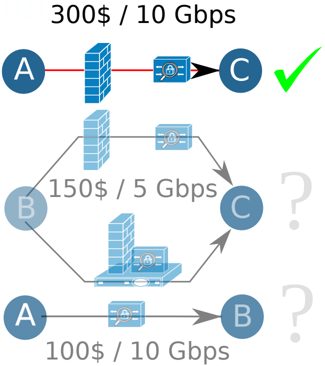

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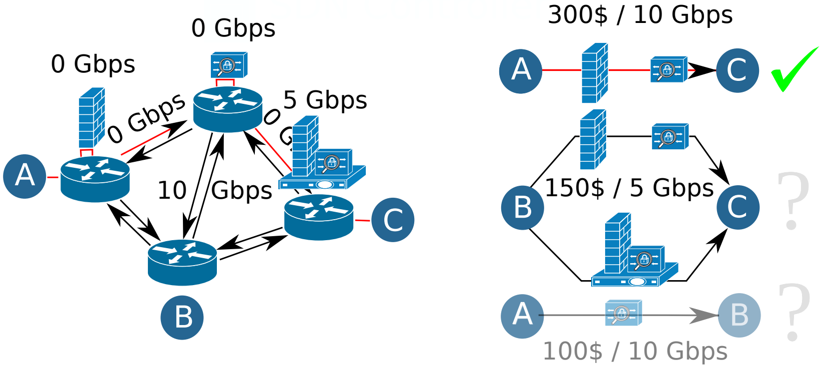

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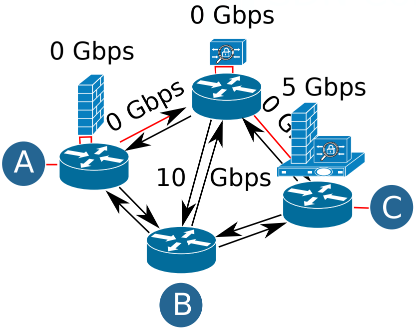

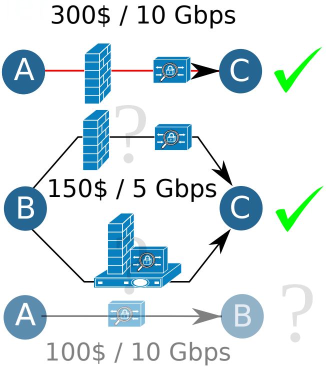

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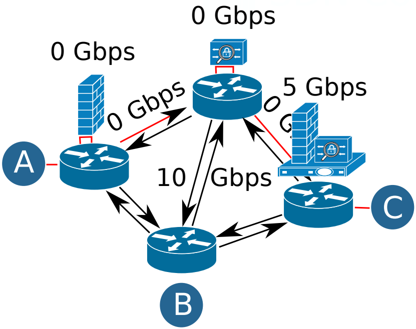

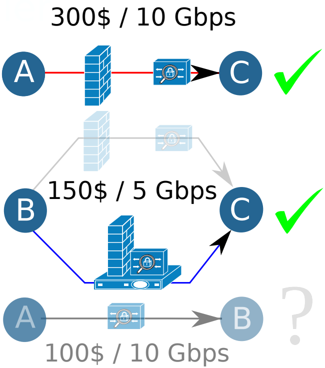

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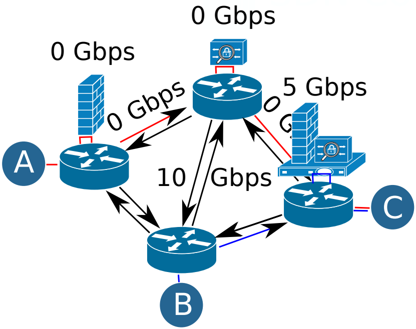

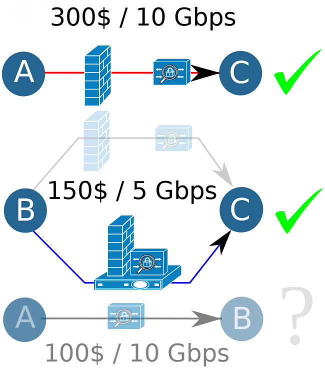

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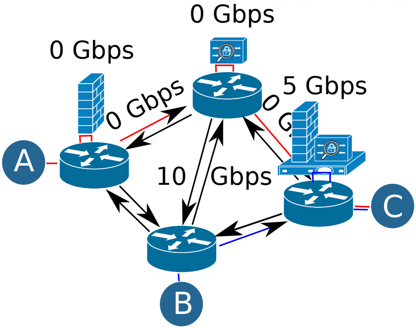

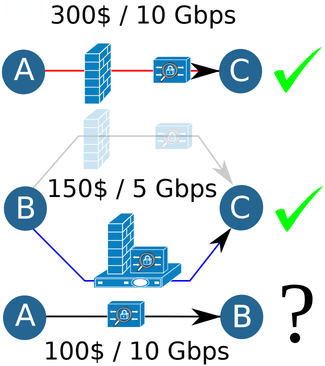

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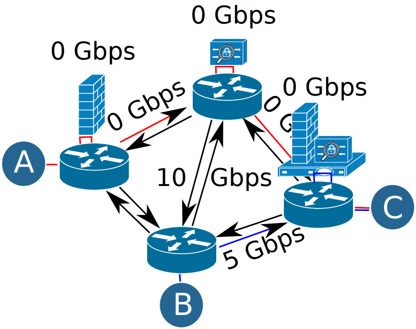

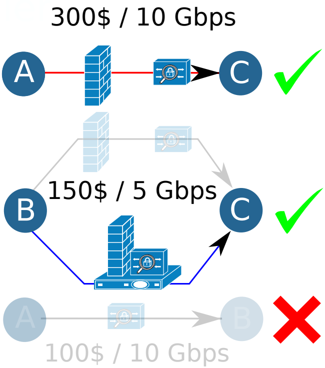

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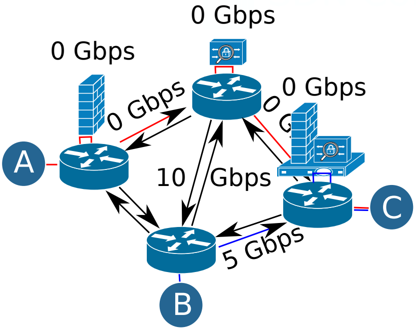
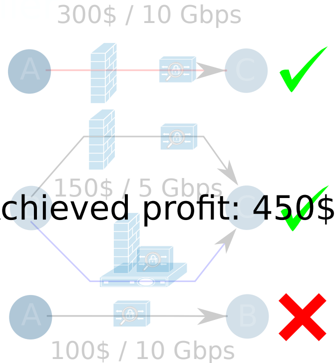




Achieved profit: 450$

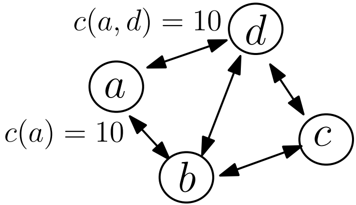
Formal Definition of PCFP

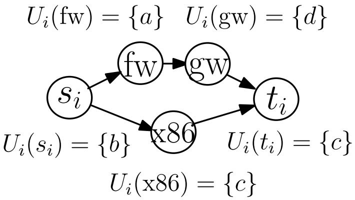
Substrate Network

- Directed network $N = (V, E)$
- capacities $c : V \cup E \to \mathbb{R}_{\geq 0}$

Requests

- Acyclic graph $G_i = (X_i, Y_i)$
- mapping restrictions $U_i : X_i \cup Y_i \to 2^V \cup 2^E$
- benefit, demand: $b_i, d_i \in \mathbb{R}_{\geq 0}$
- start, target: $s_i, t_i \in X_i$





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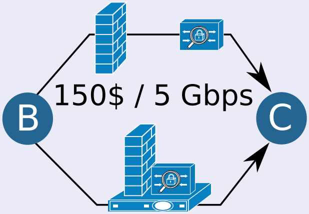

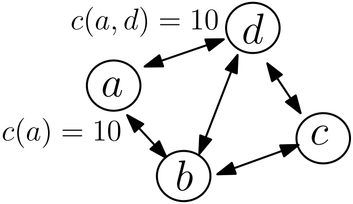

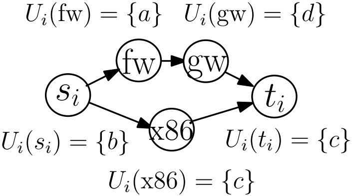

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- start, target: $s_i, t_i \in X_i$

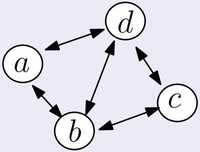
Task

Find set $I' \subseteq I$ of requests to embed and valid realizations $\tilde{p}_i$ for $i \in I'$, s.t.

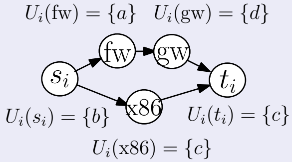
1. $\tilde{p}_i$ represents a path from $s_i \rightsquigarrow t_i$
2. capacities of substrate nodes and edges is not violated
3. the profit $\sum_{i \in I'} b_i$ is maximized.

Formal Definition of PCFP

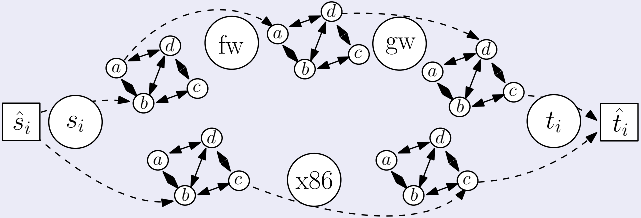
Substrate



Request r_i



Valid Realizations via Product Networks: $pn(N, r_i)$

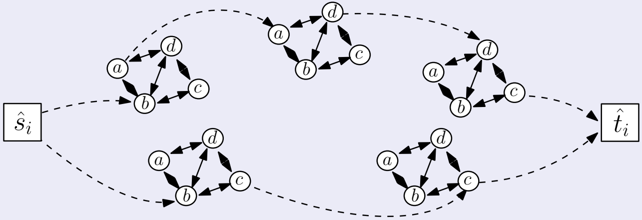


Formal Definition of PCFP

Valid Realizations

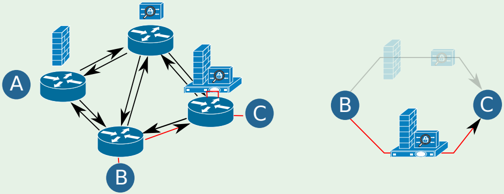
Any $\hat{s}_i$-$\hat{t}_i$ path in $pn(N, r_i)$ represents a valid realization of request r_i.

Valid Realizations via Product Networks: $pn(N, r_i)$

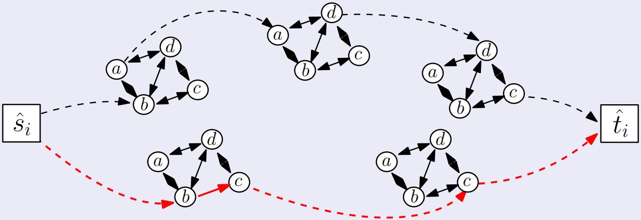


Formal Definition of PCFP

Valid Realizations: Example 1

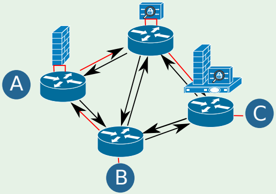


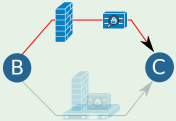
Valid Realizations in Product Networks $pn(N, r_i)$



Formal Definition of PCFP

Valid Realizations: Example 2





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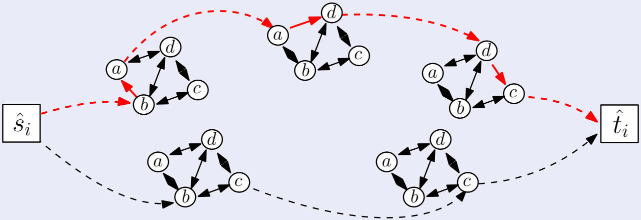
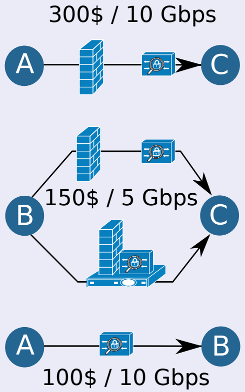


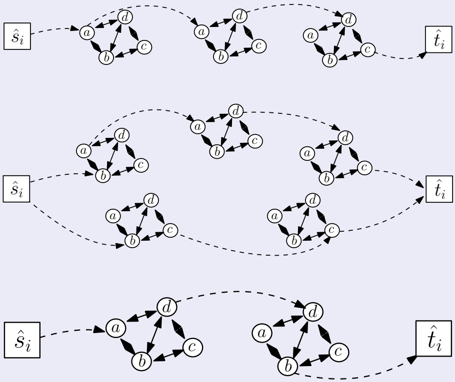
Approximating PCFP

PCFP as a Flow Problem

Requests



Product Networks

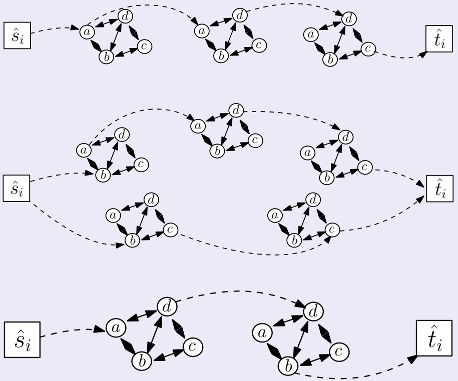


PCFP as a Flow Problem

Flow Formulation

- Compute *unsplittable* flows $\tilde{f}_i : E(pn(N, r_i)) \to \{0, d_i\}$
- Flow preservation within each product network (except at $\hat{s}_i$ and $\hat{t}_i$)
- $\max \sum_i b_i \cdot |\tilde{f}_i| / d_i$
- s.t. node and edge capacities are not violated

Product Networks



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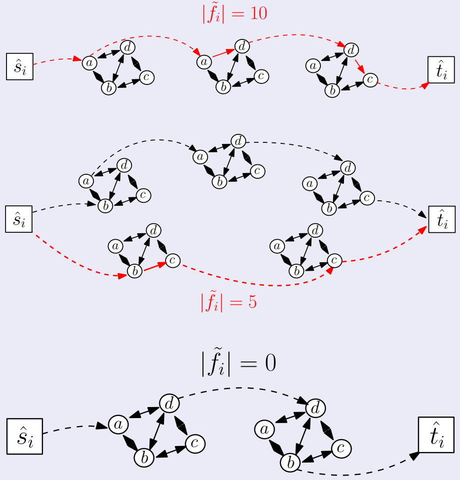
Flow Solution in Product Networks



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NP-Hardness follows from ...

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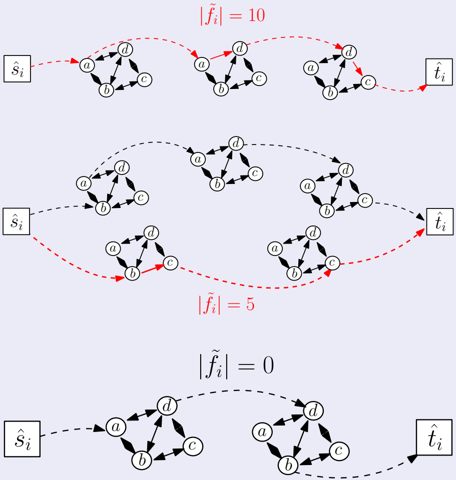
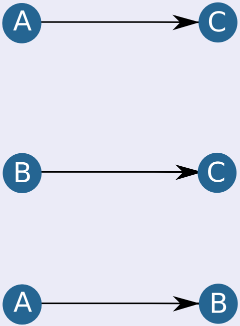
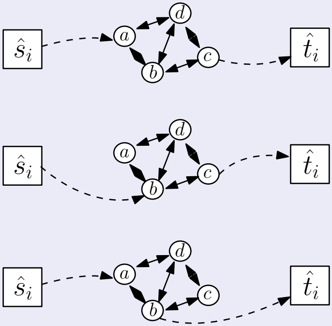


PCFP as a Flow Problem

USF Requests



Product Networks



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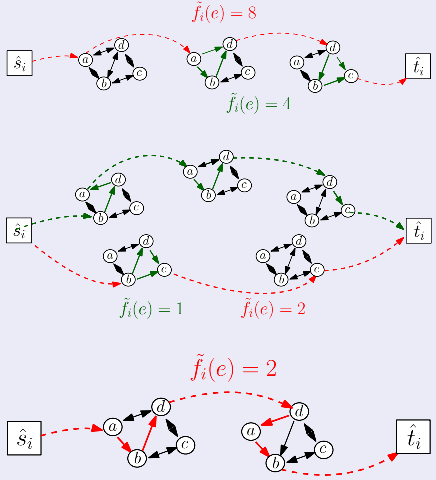
Approximating PCFP using Randomized Rounding: Idea

Flow Formulation

- Compute flows as above, but relax integrality: $\tilde{f}_i : E(pn(N, r_i)) \to [0, d_i]$

Product Networks



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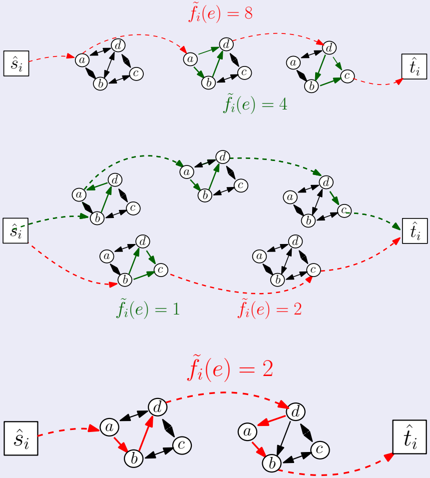
Algorithm

1. Scale capacities by $1/(1+\varepsilon)$
2. Compute fractional flows
3. Place request $i \in I$ into set $I' \subseteq I$ with probability $|\tilde{f}_i|/d_i$
4. Perform random walks to obtain $\tilde{p}_i$ for $i' \in I'$

Product Networks



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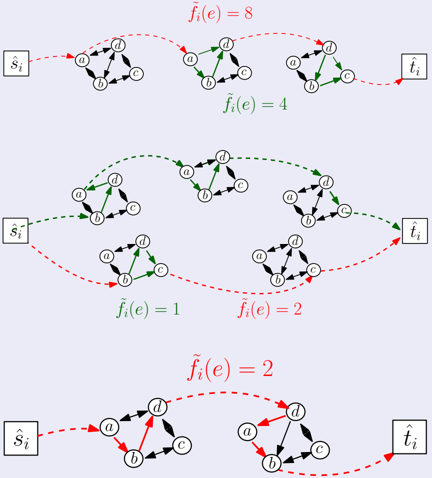


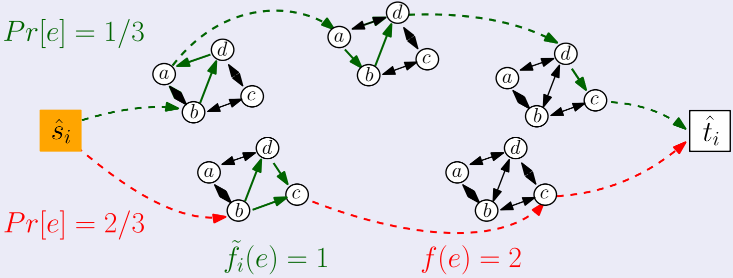
Questions

1. What is the expected profit?
2. How badly do we violate capacities?

Performing Random Walks

Approximating PCFP using Randomized Rounding: Random Walk

Random Walk at node v: $Pr[\text{choose } e] = \tilde{f}_i(e) / \sum_{e' \in \delta^+(v)} \tilde{f}_i(e)$



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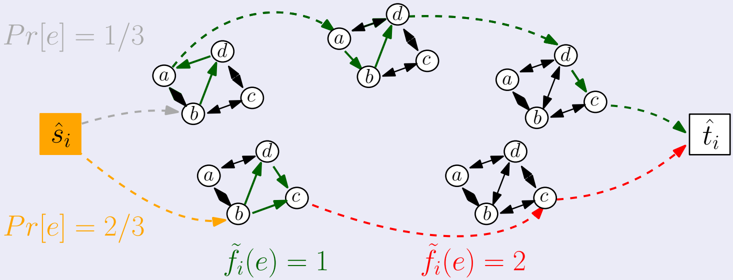

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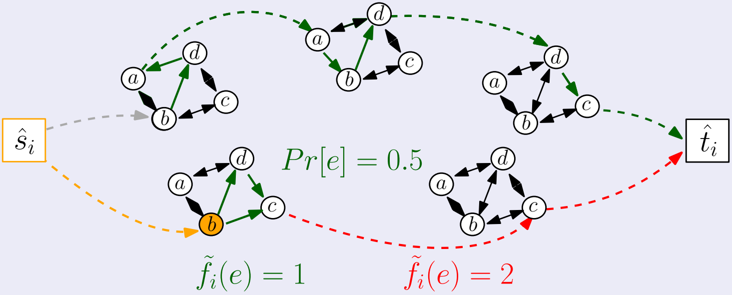

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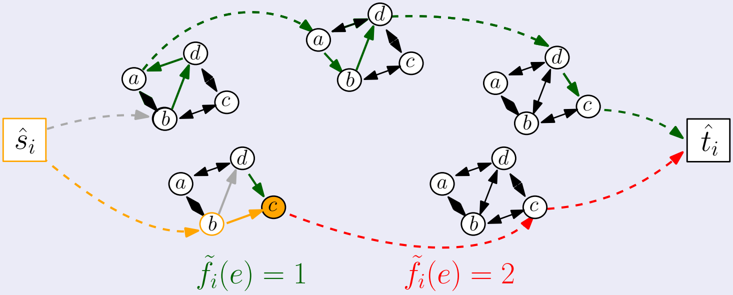

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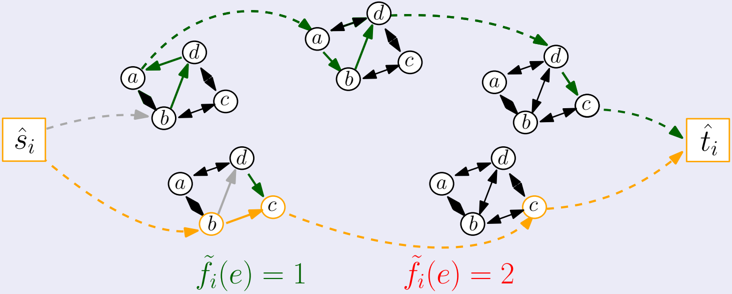
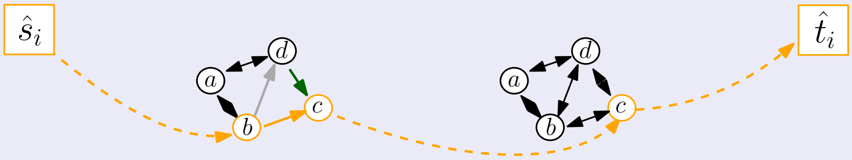


Theorem (by induction, cf. Motwani et al. [1996])

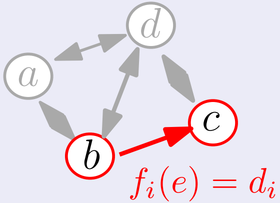
The probability that an edge $e \in E(pn(N, r_i))$ will be used equals $\tilde{f}_i(e)/d_i$.
Hence, the expected load on an edge $e \in E(pn(N, r_i))$ equals $\tilde{f}_i(e)$.

Approximating PCFP using Randomized Rounding: Random Walk

Resulting realization $\tilde{p}_i$

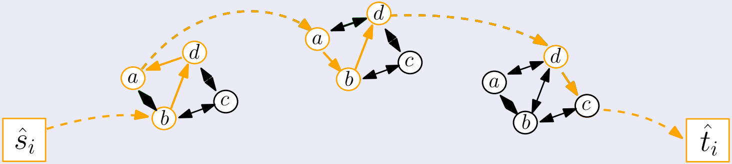


Projected flow f_i



Approximating PCFP using Randomized Rounding: Random Walk

Other potential realization $\tilde{p}_i$



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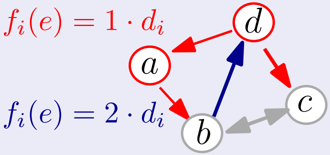

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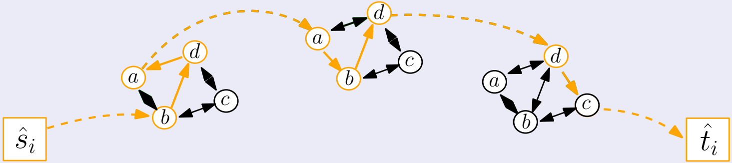
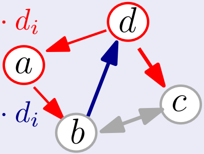


Projected flow f_i

$f_i(e) = 1 \cdot d_i$

$f_i(e) = 2 \cdot d_i$



Notation

Let $E_i(e)$ denote all *copies* of edge $e \in E$ within $pn(N, r_i)$.

Important

$f_i(e) \leq |E_i(e)| \cdot d_i$.

Analysis of Randomized Rounding

Approximating PCFP using Randomized Rounding: Analysis of Edge Capacities

- Let $\Delta_{\max} = \max_{i \in I} E_i(e)$ and $d_{\max} = \max_{j \in I} d_i$

Approach: Fix single substrate edge $e \in E$

- Interpret projected flow $f_i(e)$ as random variable.
- Request i's allocation is $X_i \triangleq f_i(e)$ with $X_i \in [0, \Delta_{\max} \cdot d_{\max}]$.

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- The capacity along edge $e \in E$ is violated, if

$$X \geq c(e) = (1 + \varepsilon) \cdot \tilde{c}(e)$$

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- Observe $\mathbf{E}[X_i] = \sum_{e' \in E_i(e)} \tilde{f}_i(e')/(\Delta_{\max} \cdot d_{\max}) \le \mu_i$ with
  - $\mu_i \triangleq \frac{\tilde{c}(e)}{(\Delta_{\max} \cdot d_{\max})} \cdot \frac{\sum_{e' \in E_i(e)} \tilde{f}_i(e')}{\sum_{j \in I} \sum_{e' \in E_j(e)} \tilde{f}_j(e')}$
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Rescaling by $1/(\Delta_{\max} \cdot d_{\max})$

Excursion: A Chernoff-Bound

Chernoff

Let $\{X_i\}_i$ denote a sequence of independent random variables attaining values in $[0,1]$. Assume that $\mathbf{E}[X_i] \leq \mu_i$. Let $X \triangleq \sum_i X_i$ and $\mu \triangleq \sum_i \mu_i$. Then, for $\varepsilon > 0$,

$$\mathbf{Pr}[X \geq (1+\varepsilon) \cdot \mu] \leq e^{-\beta(\varepsilon) \cdot \mu}.$$

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Definition of β

The function $\beta : (-1,\infty) \to \mathbb{R}$ is defined by $\beta(\varepsilon) \triangleq (1+\varepsilon)\ln(1+\varepsilon) - \varepsilon$.

Observation

For $0 < \varepsilon < 1$ we have $\beta(\varepsilon) \ge \frac{2\varepsilon^2}{4.2+\varepsilon}$ and hence $\beta(\varepsilon) = \Theta(\varepsilon^2)$.

Approximating PCFP using Randomized Rounding: Analysis of Edge Capacities

Approach: Fix single substrate edge $e \in E$

- Define $X_i \in [0,1]$: $X_i \triangleq f_i(e)/(\Delta_{\max} \cdot d_{\max})$, with $\mathbf{E}[X_i] \leq \mu_i$.
- Let $X = \sum_{i \in I} X_i$ with $\mathbf{E}[X] \leq \mu = \tilde{c}(e)/(\Delta_{\max} \cdot d_{\max})$.
- The capacity along edge $e \in E$ is violated, if $X \geq (1+\varepsilon) \cdot \mu$

Application of Chernoff-Bound

$$\mathbf{Pr}\left[\sum_{i \in I} X_i \geq (1+\varepsilon) \cdot \mu\right] \leq e^{-\beta(\varepsilon) \cdot \mu} = e^{-\beta(\varepsilon) \cdot \tilde{c}(e)/(\Delta_{\max} \cdot d_{\max})}$$

Under small demands, i.e. assuming $\frac{\tilde{c}(e)}{\Delta_{\max} d_{\max}} \geq \frac{4.2+\varepsilon}{\varepsilon^2} \cdot \ln |E|$

As $\beta(\varepsilon) \geq \dfrac{2\varepsilon^2}{4.2+\varepsilon}$ holds, $\mathbf{Pr}\left[\sum_{i \in I} X_i \geq (1+\varepsilon) \cdot \mu\right] \leq 1/|E|^2$ follows.

Main Results

Approximating PCFP using Randomized Rounding: Main Results

Main Theorem

Assume that $\frac{c_{\min}}{\Delta_{\max} \cdot d_{\max}} \geq \frac{4.2+\varepsilon}{\varepsilon^2} \cdot (1+\varepsilon) \cdot \ln|E|$ for $\varepsilon \in (0,1)$. The rounding scheme – under scaling capacities by $1/(1+\varepsilon)$ – yields

$$\mathbf{Pr}\left[\text{original edge capacity is violated}\right] \leq \frac{1}{|E|}$$

$$\mathbf{Pr}\left[B(\text{alg}) < \frac{1-\varepsilon}{1+\varepsilon} \cdot B(\text{opt}^*)\right] \leq e^{-\beta(-\varepsilon) \cdot B(\text{opt}^*)/((1+\varepsilon) \cdot b_{\max} \cdot d_{\max})}.$$

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Las Vegas

By repeating the rounding finitely many times, a high quality solution can be found with high probability.

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Corollary

If additionally, $b_i = 1$ holds for all $i \in I$, then with probability $1 - O(1/Poly(|E|)$, the algorithm returns a solution with at least $1 - O(\varepsilon)$ times the optimal benefit with high probability.

Conclusion

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Summary

- PCFP considers the placement of functions and the routing between these for multiple requests to maximize the profit.
- Apply randomized rounding (cf. Raghavan and Tompson [1987]) and obtain approximation under certain assumptions:
  - Small demands $\frac{\tilde{c}(e)}{\Delta_{\mathbf{max}} d_{\mathbf{max}}} \geq \frac{4.2+\varepsilon}{\varepsilon^2} \cdot \ln |E|$ to not violate capacites
  - Small demands and unit benefits yield $1 - \mathcal{O}(\varepsilon)$ approximation.

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  - Small demands and unit benefits yield $1 - \mathcal{O}(\varepsilon)$ approximation.

Contribution: "Rediscovery" of randomized rounding

- Consider several (virtual) embedding options for requests (DAGs).
- Show applicability of randomized rounding to exert admission control.
- Perform concise mathematical analysis.
- *First non-trivial approximation for embedding of multiple graphs.*

Related Work

Randomized Rounding

- VLSI design to minimize width [Raghavan and Tompson, 1987]
- Analysis of the approximation for PCFP without requiring assumptions and generalization to 'cyclic' requests [Rost and Schmid, 2016]

Modeling and Embedding Requests

- Product Network and Online Approximation [Even et al., 2016]
- Heuristics for choosing virtual embedding options and embedding services [Sahhaf et al., 2015]

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