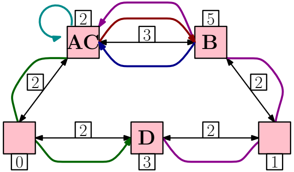
Approximate Graph Embeddings in the Cloud

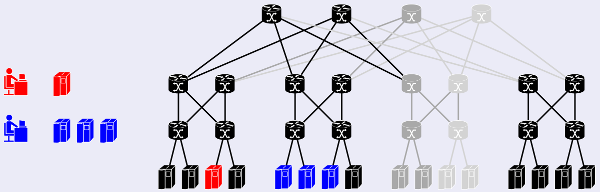


Highlights of Algorithms 2018

Matthias Rost
Technische Universität Berlin, Internet Network Architectures

Stefan Schmid
Universität Wien, Communication Technologies

Cloud Providers Offer Data Center Resources



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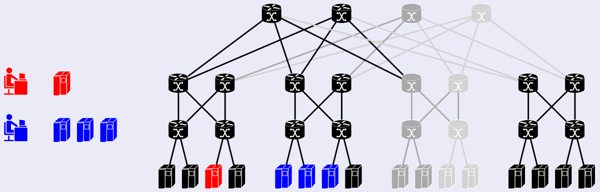

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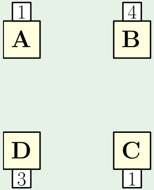

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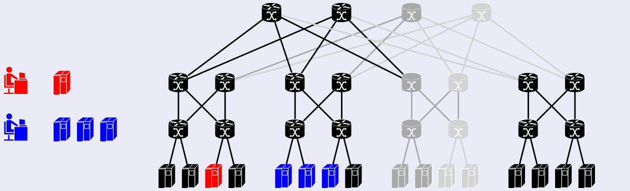

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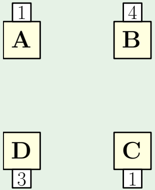

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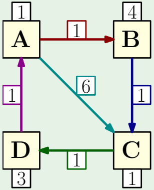

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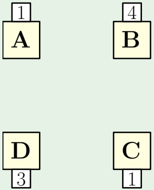

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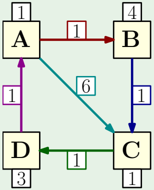

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- Map virtual nodes to substrate nodes
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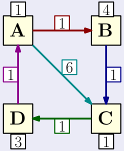
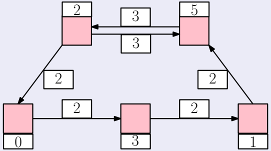
Virtual Network



Substrate (Physical Network)



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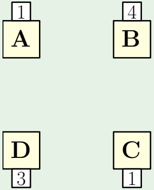

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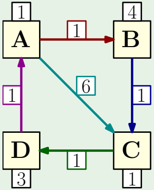

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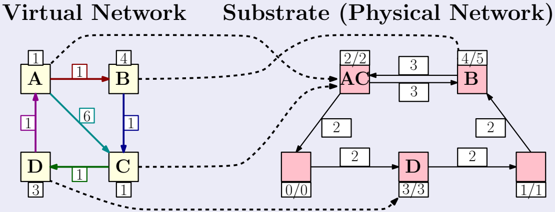

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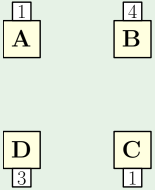

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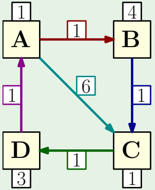

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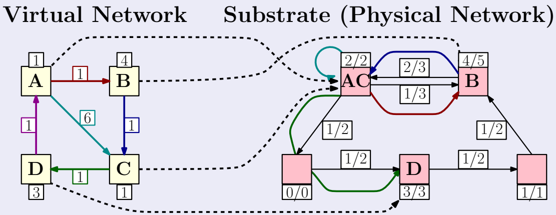

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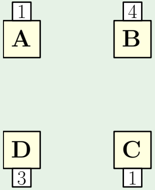

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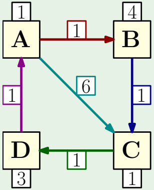

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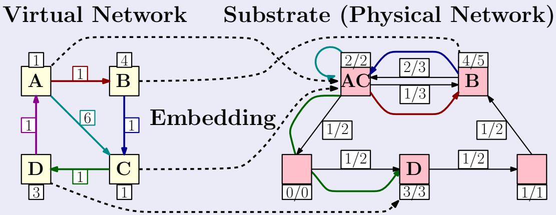

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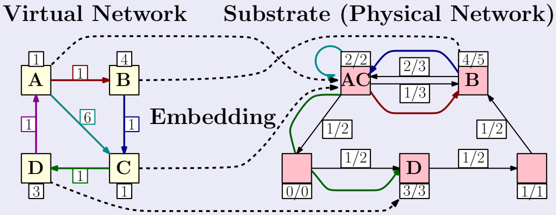

Related Work

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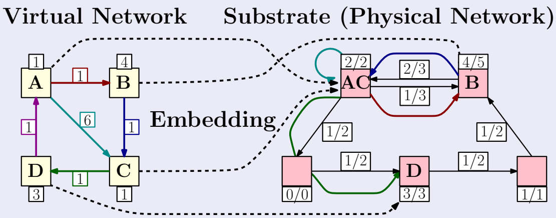

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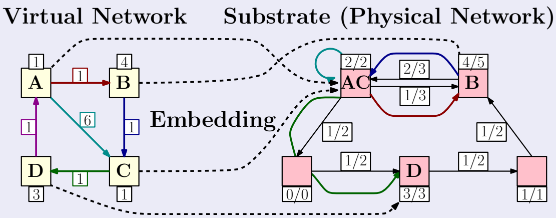

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Setting: Multiple Virtual Network requests are given

Objectives: Maximize profit (admission control) or minimize 'cost' *s.t. capacity constraints*.

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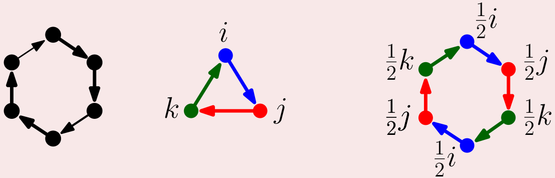

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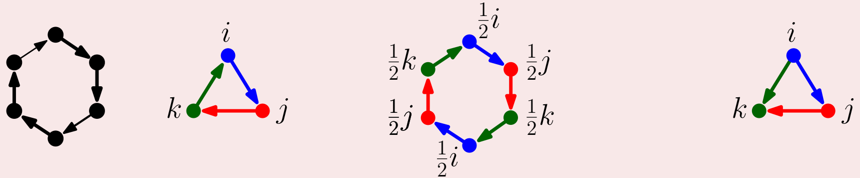

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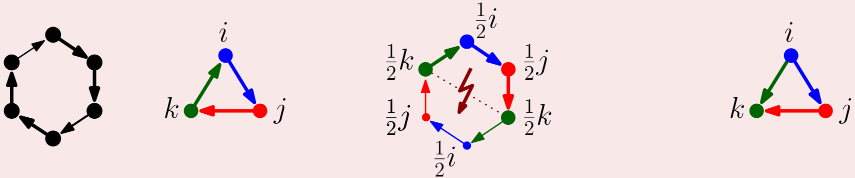

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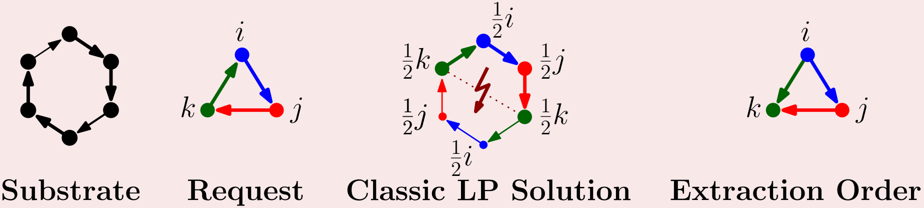

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