Beyond the Stars: Revisiting Virtual Cluster Embeddings

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Joint work with *Carlo Fuerst, Stefan Schmid*
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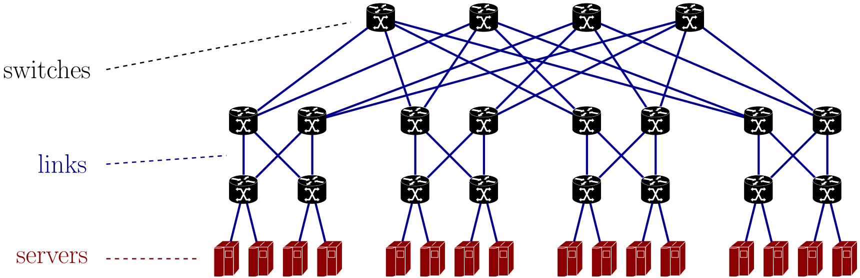
Providing Services inside Data Centers

- Example fat tree data center topology [1]
- 2.5k switches and 27k hosts for a medium sized data center



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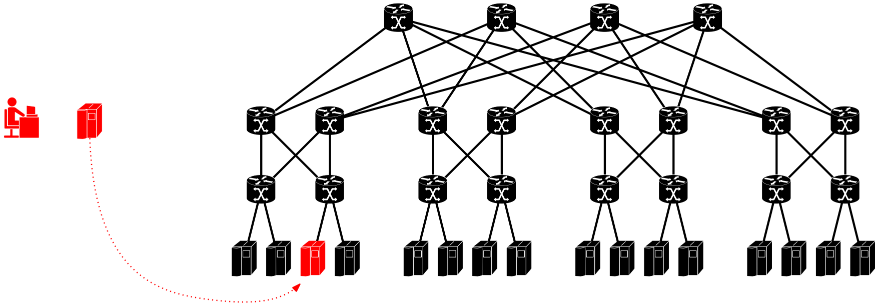
- Virtualization of servers allows to quickly spawn Virtual Machines (VMs) for tenants inside the data center



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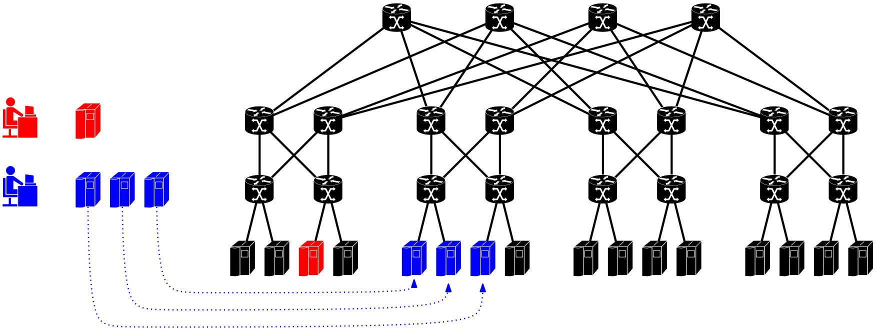
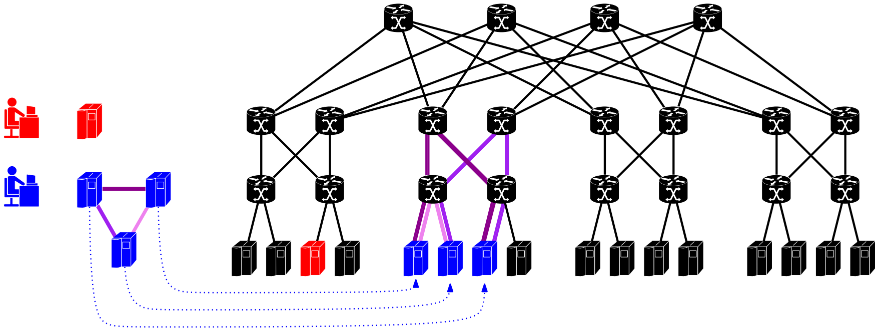
- Hundreds or thousands of Virtual Machines may be requested



Providing Services inside Data Centers

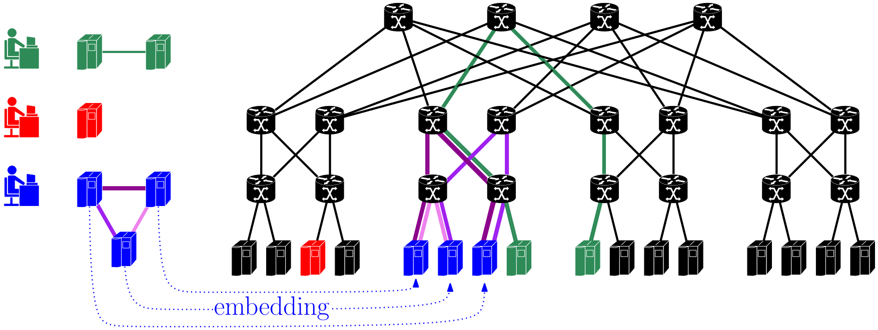
- Virtualization of servers allows to quickly spawn Virtual Machines (VMs) for tenants inside the data center
- Hundreds or thousands of Virtual Machines may be requested
- Working together, communication between VMs is of paramount importance



Providing Services inside Data Centers

Problem: Performance crucially depends on bandwidth

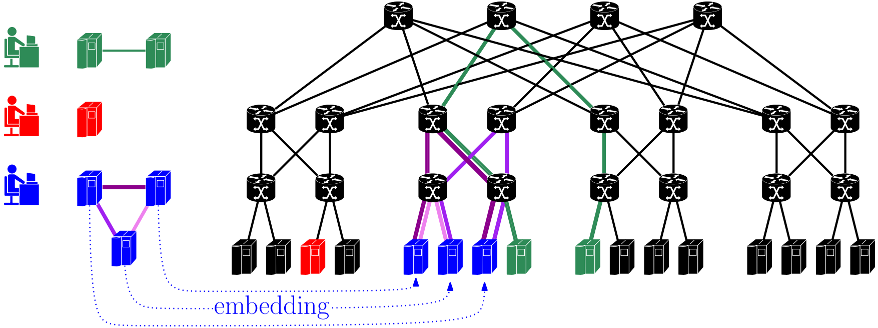
- Network transfers: 33% of the execution time (Facebook [4])
- Data centers exhibit oversubscription factors of up to 1:240 [6]
- Customer's performance varies dramatically depending on network load



Providing Services inside Data Centers

Problem: Performance crucially depends on bandwidth

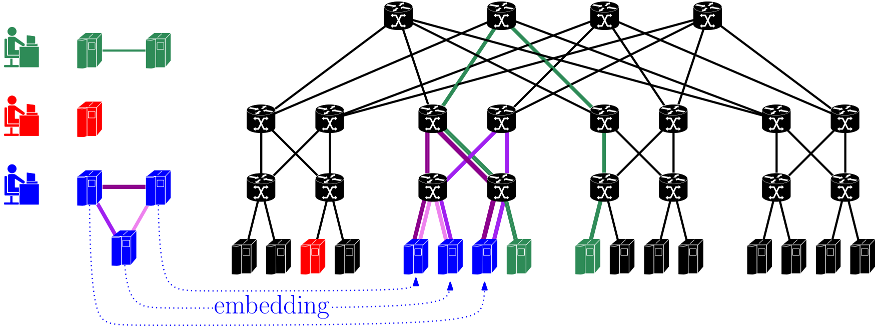
Solution: resource isolation / Quality-of-Service



Providing Services inside Data Centers

Algorithmic Task: Graph Embedding

- find embedding, i.e. a joint mapping of VMs to servers and VM interconnections to paths
- not exceeding the data center's resource capacities and of minimal cost

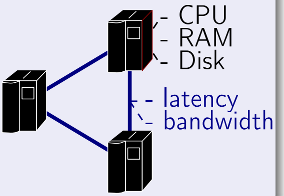


Service Abstractions: The VC Abstraction

Service Abstractions

Early 2000s: Virtual Network Embedding Problem

- Requests are specified as graphs
  - Nodes represent VMs
  - Edges represent inter-VM links



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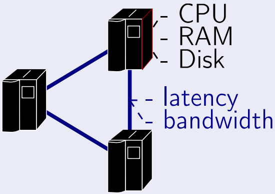

Pro

- *Concise specification*

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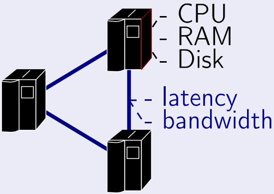


Pro

- *Concise specification*

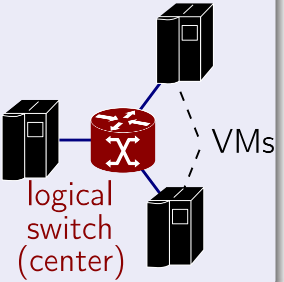
Contra

- Do customers know their requirements?
- Generally: Challenging NP-hard problem!

The Right Level of Abstraction

2011: Virtual Cluster (VC) Abstraction [3]

- Allows only for 'star'-shaped graphs
- VMs are connected to logical switch



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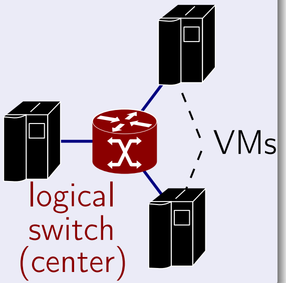

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- Allows only for 'star'-shaped graphs
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  $\mathcal{N} \in \mathbb{N}$ number of virtual machines
  $\mathcal{C} \in \mathbb{N}$ size of virtual machines
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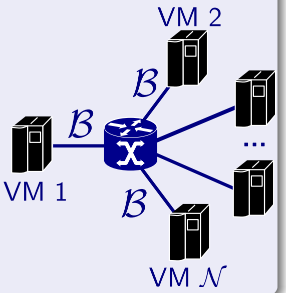

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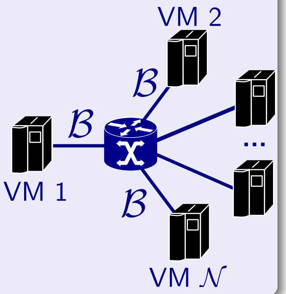


Pro

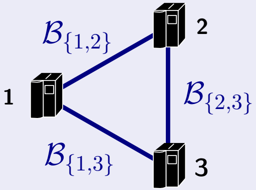
- *Simple specification!*
- *Well-performing heuristics for data-center topologies [3, 8]*

Contra

- The VM size and the amount of bandwidth are dictated by the maximum → wasteful

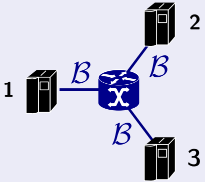
On Traffic Matrices

Graph Abstraction



- Allows for any traffic matrix M, where the bandwidth for edge $\{i, j\}$ is less than $\mathcal{B}_{\{i,j\}}$.

VC Abstraction



- Allows for any traffic matrix M, where for any VM the sum of outgoing and incoming traffic is less than $\mathcal{B}$

Outlook

Previous works . . .

- only considered (fat) trees
- only considered heuristics

Ballani et al.: ‘Oktopus’ [3]

“allocating virtual cluster requests on graphs with bandwidth-constrained edges is NP-hard”

Xie et al.: ‘Proteus’ [8]

“[Our algorithm] picks the first fitting lowest-level subtree out of all such lowest-level subtrees.”

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Main Questions

Is the VC embedding problem really NP-hard to solve?

Formal Definition of the VC Embedding Problem

VC Embedding Problem Defintion

VC request: $\mathcal{N}, \mathcal{B}, \mathcal{C}$

- $\mathsf{VC} = (V_{\mathsf{VC}}, E_{\mathsf{VC}})$,
- $V_{\mathsf{VC}} = \{1, 2, \ldots, \mathcal{N}, \mathsf{center}\}$
- $E_{\mathsf{VC}} = \{\{i, \mathsf{center}\} | 1 \leq i \leq \mathcal{N}\}$

Physical Network (Substrate)

- $\mathsf{S} = (V_{\mathsf{S}}, E_{\mathsf{S}}, \mathsf{cap}, \mathsf{cost})$,
- $\mathsf{cap} : V_{\mathsf{S}} \cup E_{\mathsf{S}} \to \mathbb{N}$
- $\mathsf{cost} : V_{\mathsf{S}} \cup E_{\mathsf{S}} \to \mathbb{R}_{\geq 0}$

Task: Find a mapping of ...

- VMs onto substrate nodes $\mathsf{map}_V : V_{\mathsf{VC}} \to V_{\mathsf{S}}$, and
- VC edges onto paths in the substrate $\mathsf{map}_E : E_{\mathsf{VC}} \to \mathcal{P}(E_{\mathsf{S}})$

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  1. $\mathsf{map}_E(\{u, v\})$ connects $\mathsf{map}_V(u)$ and $\mathsf{map}_V(v)$ for $\{u, v\} \in E_{\mathsf{VC}}$

VC Embedding Problem Defintion

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1. $\mathsf{map}_E(\{i, j\})$ connects $\mathsf{map}_V(i)$ and $\mathsf{map}_V(j)$ for $\{i.j\} \in E_{\mathsf{VC}}$
2. $\sum_{\substack{v' \in V_{\mathsf{VC}} \setminus \{\mathsf{center}\} \\ v = \mathsf{map}_V(v')}} \mathcal{C} \leq \mathsf{cap}(v)$ and $\sum_{\substack{e' \in E_{\mathsf{VC}} \\ e \in \mathsf{map}_E(e')}} \mathcal{B} \leq \mathsf{cap}(e)$ for $v \in V_{\mathsf{S}}, e \in E_{\mathsf{S}}$

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3. minimizing the cost $\mathcal{C} \cdot \sum_{v \in V_{\text{VC}} \setminus \{\text{center}\}} \text{cost}(\text{map}_V(v)) + \mathcal{B} \cdot \sum_{\substack{e' \in E_{\text{VC}} \\ e \in \text{map}_E(e')}} \text{cost}(e)$.

VC-ACE Algorithm

Key Insights

Lemma

We can assume $\mathcal{B} = \mathcal{C} = 1$.

Proof idea.

If $\mathcal{B} \neq 1$, $\mathcal{C} \neq 1$, we transform the substrate by scaling capacities and costs:

- $\text{cap}_{S'}(u) = \lfloor \text{cap}(u)/\mathcal{C} \rfloor$ for $u \in V_S$
- $\text{cap}_{S'}(e) = \lfloor \text{cap}(e)/\mathcal{B} \rfloor$ for $e \in E_S$
- $\text{cost}_{S'}(u) = \text{cost}(u) \cdot \mathcal{C}$ for $u \in V_S$
- $\text{cost}_{S'}(e) = \text{cost}(e) \cdot \mathcal{B}$ for $e \in E_S$

□

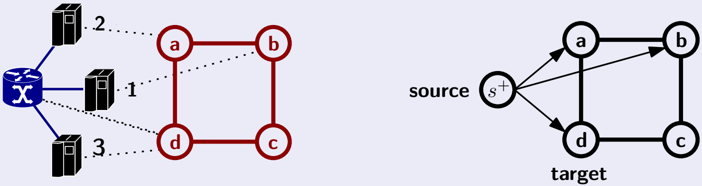
Key Insights

Lemma

We can solve the edge embedding problem if all nodes are placed.

Proof.

1. Construct extended graph with additional node s^+ and (parallel) edges: $\{(s^+, \text{map}_V(i)) | i \in \{1, \ldots, \mathcal{N}\}\}$ of capacity 1 and cost 0
2. Compute a minimum cost flow of value $\mathcal{N}$ from s^+ to $\text{map}_V(\text{center})$.
3. Perform a path-decomposition to obtain mapping for edges.



□

Key Insights

Lemma

We can solve the embedding problem if the logical switch is placed.

Proof.

1. Construct extended graph with additional edges $\{(s^+, u) | u \in V_S\}$, $\text{cap}(s^+, u) = \text{cap}(u)$ and $\text{cost}(s^+, u) = \text{cost}(u)$ for $u \in V_S$
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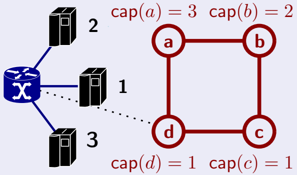
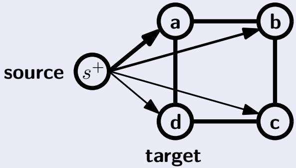
3. Perform path-decomposition to obtain mapping for *nodes and edges*.





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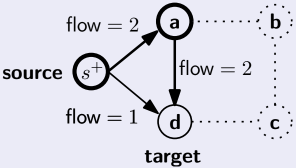

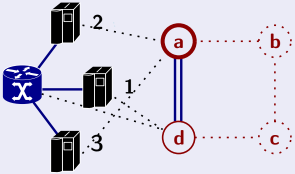

□

VC-ACE Algorithm

Idea

Simply iterate over possible locations for the center.

Algorithm 1: The VC-ACE Algorithm

Input: Substrate $S = (V_S, E_S)$, request $(\mathcal{N}, \mathcal{B}, \mathcal{C})$

Output: Optimal VC mapping $\text{map}_V, \text{map}_E$ if feasible

$(\hat{f}, \hat{v}) \leftarrow (\texttt{null}, \texttt{null})$

for $v \in V_S$ **do**

$V_{S'} = V_S \cup \{s^+\}$ **and**

$E_{S'} = E_S \cup \{(s^+, u) | u \in V_S\}$

$\text{cap}_{S'}(e) = \begin{cases} \lfloor \text{cap}(e)/\mathcal{B} \rfloor & \text{, if } e \in E_S \\ \lfloor \text{cap}(u)/\mathcal{C} \rfloor & \text{, if } e = (s^+, u) \in E_S \end{cases}$

$\text{cost}_{S'}(e) = \begin{cases} \text{cost}(e) \cdot \mathcal{B} & \text{, if } e \in E_S \\ \text{cost}(u) \cdot \mathcal{C} & \text{, if } e = (s^+, u) \in E_S \end{cases}$

$f \leftarrow$ $\texttt{MinCostFlow}(s^+, v, \mathcal{N}, V_{S'}, E_{S'}, \text{cap}_{S'}, \text{cost}_{S'})$

if f **is feasible** and $\text{cost}(f) < \text{cost}(\hat{f})$ **then**

$(\hat{f}, \hat{v}) \leftarrow (f, v)$

if $\hat{f} = \texttt{null}$ **then**

return $\texttt{null}$

return $\texttt{DecomposeFlowIntoMapping}(\hat{f}, \hat{v})$

VC-ACE Algorithm

Idea

Simply iterate over possible locations for the center.

Theorem

Correctness follows from the lemma on the previous slide.

Algorithm 2: The VC-ACE Algorithm

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VC-ACE Algorithm

Theorem

The runtime is $\mathcal{O}\left(\mathcal{N}(n^2 \log n + n \cdot m)\right)$ with $n = |V_S|$ and $m = |E_S|$, when using the successive-shortest path for the flow computation.

Corollary.

The VC Embedding Problem can be solved optimally in polynomial time. □

Algorithm 3: The VC-ACE Algorithm

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Output: Optimal VC mapping $\text{map}_V, \text{map}_E$ if feasible

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We can compute optimal solutions in polynomial-time.

Can we do even better?

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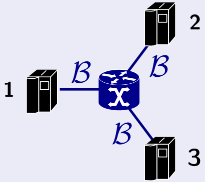
Can we do even better?

Introducing the Hose-Based Virtual Cluster

Starting from Scratch

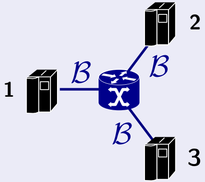
VC Abstraction



- Allows for any traffic matrix M, where for any VM the sum of outgoing and incoming traffic is less than $\mathcal{B}$

Starting from Scratch

VC Abstraction



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Question:

What is the purpose of the switch?

Starting from Scratch

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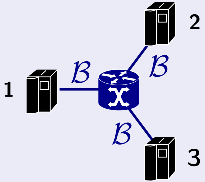

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Question:

What is the purpose of the switch?

Ballani et al. 'Oktopus' [3]

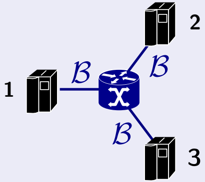
"Oktopus' allocation algorithms assume that the traffic between a tenant's VMs is routed along a tree."

Answer:

To route the traffic along a tree.

Starting from Scratch

VC Abstraction



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Can we do without the switch?

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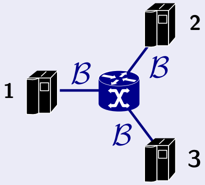

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Question:

Can we do without the switch?

Ballani et al. 'Oktopus' [3]

"Alternatively, the NM [Network Manager] can control datacenter routing to actively build routes between tenant VMs [..]"

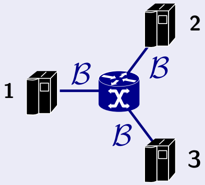
"We defer a detailed study of the relative merits of these approaches to future work."

Answer:

Yes!

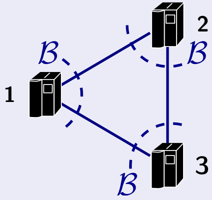
Starting from Scratch

VC Abstraction



- Allows for any traffic matrix M, where for any VM the sum of outgoing and incoming traffic is less than $\mathcal{B}$

Hose-Based VC Abstraction



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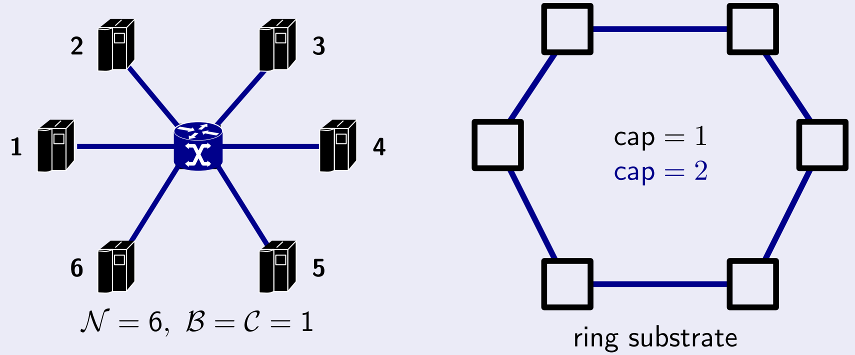
Motivating Example



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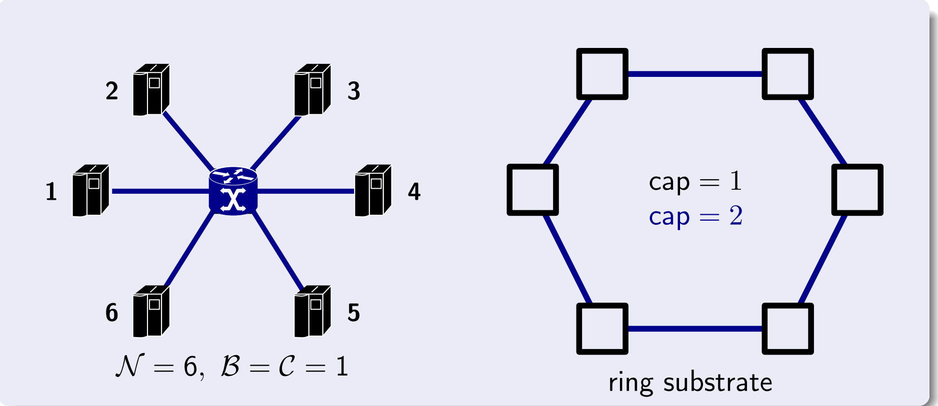
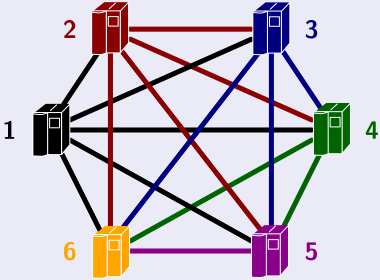


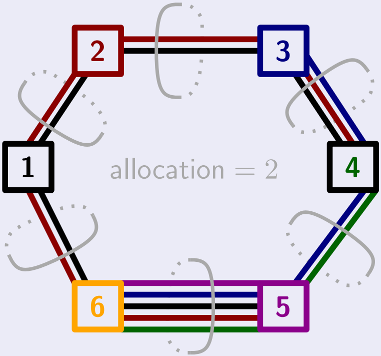
There exists no solution ...

... in the classic VC embedding model.

Motivating Example

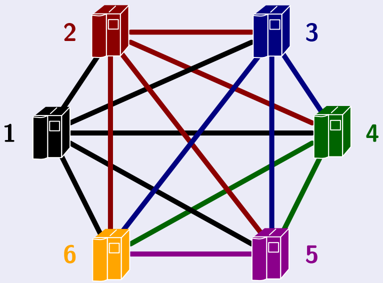


$\mathcal{N} = 6, \ \mathcal{B} = \mathcal{C} = 1$



Embedding on the same substrate

Motivating Example



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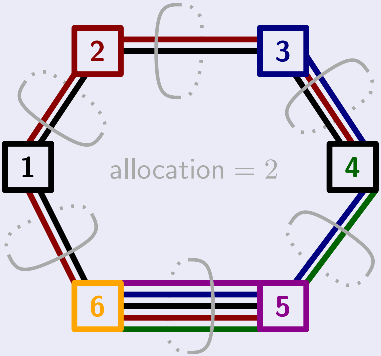
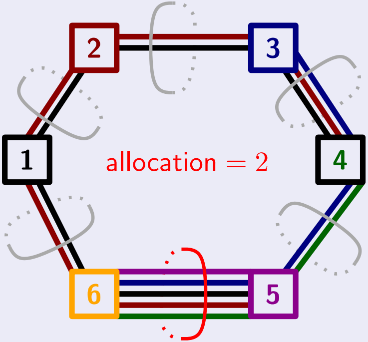


Embedding on the same substrate

There exists a solution ...

... in the hose-based VC model!

Motivating Example

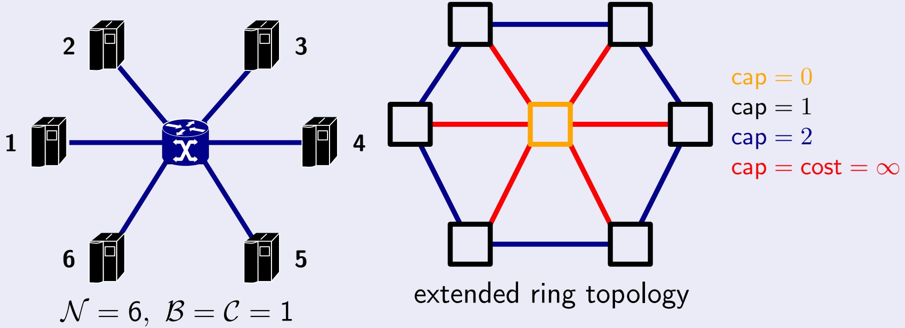


Embedding on the same substrate

Why allocations of 2 are sufficient:

- Consider edge e between VMs 6 and 5.
- The edge is used by routes $R(e) = \{(1,5),(2,5),(3,6),(4,6),(5,6)\}$.
- Any valid traffic matrix M will respect:
  - $M_{1,5} + M_{2,5} \leq 1$
  - $M_{3,6} + M_{4,6} + M_{5,6} \leq 1$
- Hence $\sum_{(i,j)\in R(e)} M_{i,j} \leq 2$ holds.

Motivating Example II



Solution costs . . .

. . . can be arbitrarily higher under the classic star-interpretation!

Hose-Based Virtual Cluster Embedding Problem

Hose-Based VC Embedding Problem (HVCEP)

Definition (Clique Graph)

- $V_C = \{1, \ldots, \mathcal{N}\}$, $E_C = \{(i,j) | i,j \in V_C, i < j\}$

Task: Find a mapping of ...

- VC nodes onto substrate nodes $\text{map}_V : V_C \to V_S$, and
- VC routes onto paths in the substrate $\text{map}_E : E_C \to \mathcal{P}(E_S)$, such that

1. *route* $(i,j) \in E_C$ connects $\text{map}_V(i)$ and $\text{map}_V(j)$,
2. the mapping of VMs must not violate node capacities (cf. slide 12),

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- VC routes onto paths in the substrate $\text{map}_E : E_C \rightarrow \mathcal{P}(E_S)$, and
- integral bandwidth reservations $l_{u,v} \leq \text{cap}(u,v)$ for $\{u,v\} \in E_S$, s.t.
  1. *route* $(i,j) \in E_C$ connects $\text{map}_V(i)$ and $\text{map}_V(j)$,
  2. the mapping of VMs must not violate node capacities (cf. slide 12),
  3. for all valid traffic matrices M – i.e. $\sum_{(i,j) \in E_C} M_{i,j} + M_{j,i} \leq \mathcal{B}$ holds – the bandwidth reservation is not exceeded on any edge $\{u,v\} \in E_S$: $\sum_{\{i,j\} \in E_C : \{u,v\} \in \text{map}_E(\{i,j\})} M_{ij} \leq l_{u,v}$,

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  4. minimizing $\mathcal{C} \cdot \sum_{i \in V_C} \text{cost}(\text{map}_V(i)) + \mathcal{B} \cdot \sum_{e \in E_S} l_{u,v} \cdot \text{cost}(e)$.

Computational Complexity of HVC Embeddings

Computational Complexity of Finding HVC Embeddings

Theorem (via the Virtual Private Network Problem [7])

Finding a feasible solution for the HVCEP is NP-hard.
This still holds if the VMs are already mapped.

Theorem (via the Virtual Private Network Conjecture [5])

Algorithm VC-ACE solves the HVCEP when capacities are sufficiently large.

Computing (Fractional) HVC Embeddings

A Mixed-Integer Programming Formulation for the HVCEP

Variables

- x_u^i mapping of VM i onto node u
- $y_{u.v}^{i,j}$ mapping of link $(i,j) \in V_C$ onto (directed) substrate edge (u,v)
- $l_{u.v}$ load on substrate edge $\{u,v\}$
- ω_{uv}^i 'dual variable' for allocation of communications of VM i on edge $\{u,v\}$

Mixed-Integer Program 1: HVC-OSPE

$$\min \sum_{i \in V_C, u \in V_S} \text{cost}_u \cdot x_u^i + \sum_{\{u,v\} \in E_S} \text{cost}_{u,v} \cdot l_{uv} \tag{1}$$

$$\sum_{u \in V_S} x_u^i = 1 \qquad \forall i \in V_C. \tag{2}$$

$$\sum_{u \in V_S} \sigma_u \cdot (x_u^i - x_u^{i+1}) \leq 0 \qquad \forall i \in V_C \setminus \{\mathcal{N}\}. \tag{3}$$

$$\sum_{i \in V_C} \mathcal{C} \cdot x_u^i \leq \text{cap}_u \qquad \forall u \in V_S. \tag{4}$$

$$l_{uv} \leq \text{cap}_{uv} \qquad \forall \{u,v\} \in E_S. \tag{5}$$

$$\sum_{(u,v) \in \delta_u^+} y_{uv}^{ij} - \sum_{(v,u) \in \delta_u^-} y_{vu}^{ij} = x_u^i - x_u^j \qquad \forall (i,j) \in E_C, \forall u \in V_S. \tag{6}$$

$$\sum_{i \in V_C} \mathcal{B} \cdot \omega_{uv}^i \leq l_{uv} \qquad \forall \{u,v\} \in E_S. \tag{7}$$

$$y_{uv}^{ij} + y_{vu}^{ij} \leq \omega_{uv}^i + \omega_{uv}^j \qquad \forall (i,j) \in E_C, \forall \{u,v\} \in E_S. \tag{8}$$

A Mixed-Integer Programming Formulation for the HVCEP

Explanation

- (2) - (4) control the VM embedding
- (5) - (8) is adapted from Altin et al. [2] for computing the optimal hose allocations on edges

Mixed-Integer Program 2: HVC-OSPE

$$\min \sum_{i \in V_C, u \in V_S} \mathrm{cost}_u \cdot x_u^i + \sum_{\{u,v\} \in E_S} \mathrm{cost}_{u,v} \cdot l_{uv} \tag{1}$$

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Observation

There are $\mathcal{O}(\mathcal{N}^2 \cdot |E_S|)$ binary variables for computing paths.

Mixed-Integer Program 3: HVC-OSPE

$$\min \sum_{i \in V_C, u \in V_S} \text{cost}_u \cdot x_u^i + \sum_{\{u,v\} \in E_S} \text{cost}_{u,v} \cdot l_{uv} \quad (1)$$

$$\sum_{u \in V_S} x_u^i = 1 \qquad \forall i \in V_C. \quad (2)$$

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$$\sum_{(u,v) \in \delta_u^+} y_{uv}^{ij} - \sum_{(v,u) \in \delta_u^-} y_{vu}^{ij} = x_u^i - x_u^j \qquad \forall (i,j) \in E_C, \forall u \in V_S. \quad (6)$$

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A Mixed-Integer Programming Formulation for the HVCEP

Observation

There are $\mathcal{O}(\mathcal{N}^2 \cdot |E_S|)$ binary variables for computing paths.

Initial Computational Results

Solving this formulation may take up to 1800 seconds for embedding a 10-VM VC onto a 20 node substrate.

Mixed-Integer Program 4: HVC-OSPE

$$\min \sum_{i \in V_C, u \in V_S} \text{cost}_u \cdot x_u^i + \sum_{\{u,v\} \in E_S} \text{cost}_{u,v} \cdot l_{uv} \tag{1}$$

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A Mixed-Integer Programming Formulation for the HVCEP

Further Observations

- The hardness result has shown that the problem is hard even if the VMs are fixed.
- The large number of variables necessary for computing each end-to-end path between VMs renders solving even the linear relaxation – i.e. dropping integrality constraints – computationally hard.

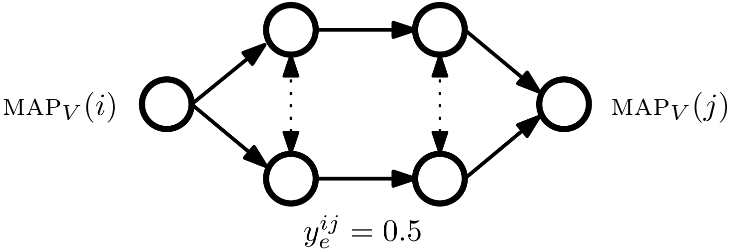
Assumptions for obtaining a 'solvable' formulation

- Assume that the VMs are already mapped.
- Assume that the hose-paths are *splittable*, i.e. each VMs are connected by a set of (weighted) paths.

Computing Splittable HVC Embeddings

Assumptions

- Assume that the VMs are already mapped.
- Assume that the hose-paths are *splittable*. arbitrarily many paths.



Computing Splittable HVC Embeddings

$$l_{uv} \leq \mathrm{cap}_{uv} \quad \forall \{u,v\} \in E_S. \quad (5)$$

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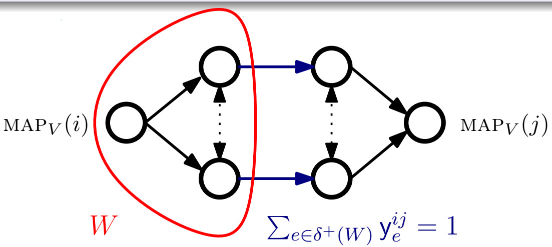

This type of constraint is equivalent to (6).

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$$\sum_{(u,v)\in\delta^+(W)} y_{uv}^{ij} \geq 1 \qquad \forall (i,j) \in E_C. \forall W \subset V_S : \text{map}_V(i) \in W, \ \text{map}_V(j) \notin W \tag{6$\star$}$$

Derivation of a new constraint

- Across a cut W, the amount of flow must be greater than 1 (6$\star$).

Computing Splittable HVC Embeddings

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- Across a cut W, the amount of flow must be greater than 1 (6⋆).
- By summing up Constraints (8) accordingly, we obtain for $(i,j) \in E_C$ and cut W that $\sum_{(u,v)\in\delta^+(W)} (\omega_{uv}^i + \omega_{uv}^j) \geq 1$ holds.

Computing Splittable HVC Embeddings

$$l_{uv} \leq \mathrm{cap}_{uv} \quad \forall \{u,v\} \in E_S. \tag{5}$$

$$\sum_{(u,v)\in\delta_u^+} y^{ij}_{uv} - \sum_{(v,u)\in\delta_u^-} y^{ij}_{vu} = x^i_u - x^j_u \quad \forall (i,j) \in E_C, \ \forall u \in V_S. \tag{6}$$

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Derivation of a new constraint

- Across a cut W, the amount of flow must be greater than 1 (6$\star$).
- By summing up Constraints (8) accordingly, we obtain for $(i,j) \in E_C$ and cut W that $\sum_{(u,v)\in\delta^+(W)} (\omega^i_{uv} + \omega^j_{uv}) \geq 1$ holds.

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Remarks

- Given (9), we can always (re-)construct the flow variables y^{ij}_{uv} afterwards by breadth-first searches.
- Furthermore, this property does not depend on (6$\star$).

Computing Splittable HVC Embeddings

$$l_{uv} \leq \text{cap}_{uv} \qquad \forall \{u,v\} \in E_S. \tag{5}$$

$$\sum_{(u,v)\in\delta_u^+} y_{uv}^{ij} - \sum_{(v,u)\in\delta_u^-} y_{vu}^{ij} = x_u^i - x_u^j \qquad \forall (i,j) \in E_C, \ \forall u \in V_S. \tag{6}$$

$$\sum_{i \in V_C} \mathcal{B} \cdot \omega_{uv}^i \leq l_{uv} \qquad \forall \{u,v\} \in E_S. \tag{7}$$

$$y_{uv}^{ij} + y_{vu}^{ij} \leq \omega_{uv}^i + \omega_{uv}^j \qquad \forall (i,j) \in E_C, \ \forall \{u,v\} \in E_S. \tag{8}$$

$$\sum_{(u,v)\in\delta^+(W)} y_{uv}^{ij} \geq 1 \qquad \forall (i,j) \in E_C . \forall W \subset V_S : \text{map}_V(i) \in W, \text{map}_V(j) \notin W \tag{6$\star$}$$

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Remarks

- Therefore Constraints (6), (8), and (6$\star$) are not needed anymore!

Computing Splittable HVC Embeddings

Algorithm 5: HMPR

$$\min \sum_{\{u,v\} \in E_S} \text{cost}_{u,v} \cdot l_{uv} \tag{10}$$

$$l_{uv} \leq \text{cap}_{uv} \quad \forall \{u,v\} \in E_S. \tag{11}$$

$$\sum_{i \in V_C} \mathcal{B} \cdot \omega^i_{uv} \leq l_{uv} \quad \forall \{u,v\} \in E_S. \tag{12}$$

$$\sum_{(u,v) \in \delta^+(W)} (\omega^i_{uv} + \omega^j_{uv}) \geq 1 \quad \forall (i,j) \in E_C . \forall W \subset V_S : \text{map}_V(i) \in W, \text{map}_V(j) \notin W \tag{13}$$

Computing Splittable HVC Embeddings

Algorithm 6: HMPR

$$\min \sum_{\{u,v\} \in E_S} \text{cost}_{u,v} \cdot l_{uv} \tag{10}$$

$$l_{uv} \leq \text{cap}_{uv} \quad \forall \{u,v\} \in E_S. \tag{11}$$

$$\sum_{i \in V_C} \mathcal{B} \cdot \omega^i_{uv} \leq l_{uv} \quad \forall \{u,v\} \in E_S. \tag{12}$$

$$\sum_{(u,v) \in \delta^+(W)} (\omega^i_{uv} + \omega^j_{uv}) \geq 1 \quad \forall (i,j) \in E_C. \forall W \subset V_S : \text{map}_V(i) \in W, \text{map}_V(j) \notin W \tag{13}$$

Exponential number of constraints, . . .

. . . which can be separated in polynomial time.

Computing Splittable HVC Embeddings

Algorithm 7: HMPR

$$\min \sum_{\{u,v\}\in E_S} \text{cost}_{u,v} \cdot l_{uv} \quad (10)$$

$$l_{uv} \leq \text{cap}_{uv} \quad \forall \{u,v\} \in E_S. \quad (11)$$

$$\sum_{i \in V_C} \mathcal{B} \cdot \omega^i_{uv} \leq l_{uv} \quad \forall \{u,v\} \in E_S. \quad (12)$$

$$\sum_{(u,v)\in\delta^+(W)} (\omega^i_{uv} + \omega^j_{uv}) \geq 1 \quad \forall (i,j) \in E_C . \forall W \subset V_S : \text{map}_V(i) \in W, \text{map}_V(j) \notin W \quad (13)$$

Exponential number of constraints, ...

...which can be separated in polynomial time.

Number of variables, ...

...in the order of $\mathcal{O}(\mathcal{N} \cdot |E_S|)$ instead of $\mathcal{O}(\mathcal{N}^2 \cdot |E_S|)$.

Computing Splittable HVC Embeddings

We can compute fractional edge embeddings, . . .

. . . but how to find node locations?

Computing Splittable HVC Embeddings

Heuristic Idea

- without capacities: "VC = HVC"
- reuse VC-ACE algorithm, but allow violation of capacities w.r.t. VC model
- violating capacities induces k times the cost of the original edge

Algorithm 5: The HVC-ACE Embedding Heuristic

Input: Substrate $S = (V_S, E_S)$,
request $\mathsf{VC}(\mathcal{N}, \mathcal{B}, \mathcal{C})$,
cost factor $k \geq 1$

Output: Splittable HVC-Embedding $\mathsf{map}_V, \mathsf{map}_E$

```
E_S' ← ∅
for e ∈ E_S do
    E_S' = E_S' ⊔ {e, e'}
    cap_S'(e) = cap(e) and cap_S'(e') = ∞
    cost_S'(e) = cost(e) and cost_S'(e') = cost(e) · k
map_V, map_E ← VC-ACE(V_S, E_S', VC(N, B, C))
if map_V ≠ null then
    map_E ← HMPR(VC(N, B, C), map_V)
    if map_E ≠ null then
        return map_V, map_E
return null
```

Computational Evaluation

What do we get by using HVC-ACE?

Setup

Topologies

- Fat trees with 12 port switches and 432 server overall
- MDCubes consisting of 4 BCubes with 12 port switches and k = 1, such that the topology contains 576 server

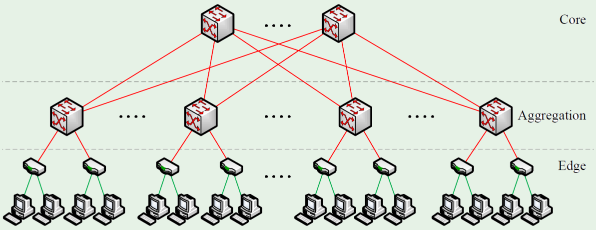


Figure : Fat tree (n=4)

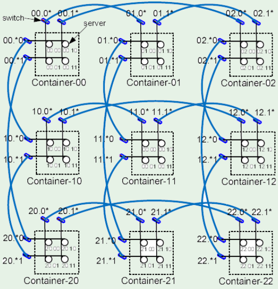


Figure : MDCube (n=2,k=1)

Setup

Generation of Requests

- $\mathcal{N}$ is chosen uniformly at random from the interval $\{10, \ldots, 30\}$.
- $\mathcal{B}$ is uniformly distributed in the interval of $\{20\%, \ldots, 100\%\}$ w.r.t. to the available capacity of an unused link.
- $\mathcal{C} = 1$ and the capacity of servers are 2.

Generation of Scenarios

- Requests are embedded over time using the VC-ACE algorithm.
- After system stabilization, a single data point is generated by considering the performance of both algorithms on the same substrate state and the same request.

Metrics

Acceptance Ratio

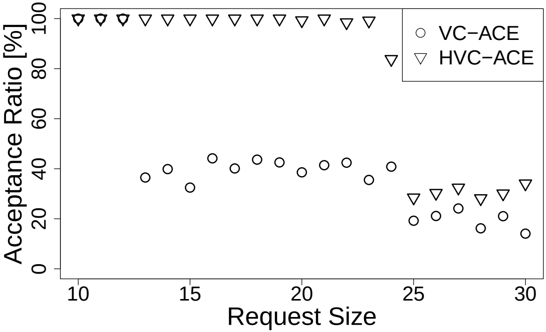
How many requests can VC-ACE embed compared to HVC-ACE?

Footprint Change

Assuming that both algorithms have found a solution, how much resources do we save by using HVC-ACE (compared to VC-ACE using 100%).

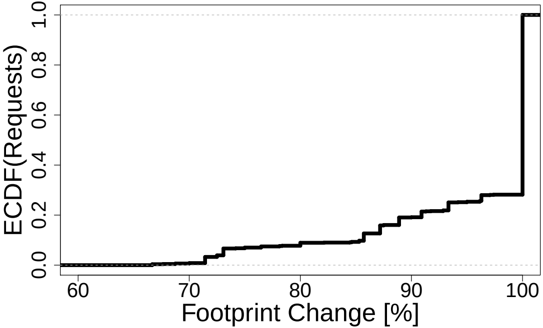
Results

Results on Fat Tree Topology



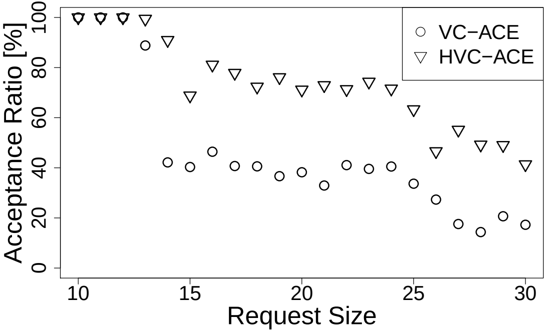
HVC-ACE can improve acceptance ratio dramatically.

Results on Fat Tree Topology



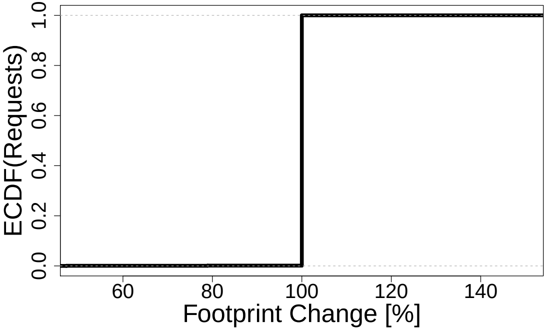
HVC-ACE saves > 10% of resources for more than 20% of the requeusts.

Results on MDCube



HVC-ACE can improve acceptance by around 20% on average.

Results on MDCube



HVC-ACE saves no resources.

Conclusion

Contributions

- Showed how to solve the classic VC embedding problem optimally.
- Defined formally the hose-based VC embedding problem and studied its computational complexity.
- Derived compact formulation for the splittable hose embedding.
- Validated that hose model can save a substantial amount of resources and increase the acceptance ratio.

Conclusion

Contributions

- Showed how to solve the classic VC embedding problem optimally.
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Bottomline

- Complexity of specification can often be traded-off with the complexity of the respective embedding algorithms.
- We need to understand this trade-off better and explore the boundaries of specifications that we can efficiently embed.

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Excursion: VPN Embeddings and the VPN Conjecture

Definition (VPN Embedding Problem (VPNEP) [7])

Given:
- Substrate network $G = (V, E)$ with edge costs $\text{cost} : E \to \mathbb{R}_0^+$
- Set of terminals $W \subseteq V$ with demands $b(i) \in \mathbb{R}^+$ for $i \in W$

Task:
- Find paths $P_{\{i,j\}}$ for all pairs $i, j \in W$, $i \neq j$, and
- bandwidth allocations $x_e \in \mathbb{R}_0^+$ on edges $e \in E$, s.t.
- $\sum_{i,j \in W: i \neq j, P_{\{i,j\}}} M_{\{i,j\}} \leq x_e$ holds for traffic matrices M,
- minimizing the cost $\sum_{e \in E} \text{cost}(e) \cdot x_e$.

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Theorem

Finding a feasible solution for the capacitated VPNEP is NP-hard [7].

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Theorem (By reduction from the VPNEP)

Finding a feasible solution for the HVCEP is NP-hard.

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Theorem (VPN Conjecture)

Tree routing and arbitrary routing solutions coincide for the VPNEP on uncapacitated graphs. [5].

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Theorem (Via the VPN Conjecture)

Algorithm VC-ACE solves the HVCEP when capacities are sufficiently large!