VirtuCast: Optimal Virtualized In-Network Processing

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March 4th, 2015, *NetAlgs Seminar*, Tel Aviv University

M.Sc. Thesis *Matthias Rost (Advisor: Stefan Schmid)*
Optimal Virtualized In-Network Processing with Applications to Aggregation and Multicast, TU Berlin '14

Conference *Matthias Rost and Stefan Schmid*
VirtuCast, Multicast and Aggregation with In-Network Processing, OPODIS '13

Tech. Report *Matthias Rost and Stefan Schmid*
The Constrained Virtual Steiner Arborescence Problem: Formal Definition, Single-Commodity Integer Programming Formulation and Computational Evaluation, arXiv '13

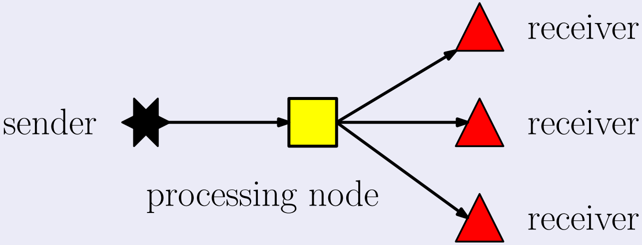
Mindset: Service Deployment

Service Deployment is not a Virtual Network Embedding

- Customer requests a ‘communication service’, but does not know how it may be embedded best
  - customer may not know the provider’s topology
  - customer may not care
- Service provider *finds* an appropriate virtual topology and embeds it

Communication Schemes: Multicast (same old! same old?)

processing = duplication + reroute



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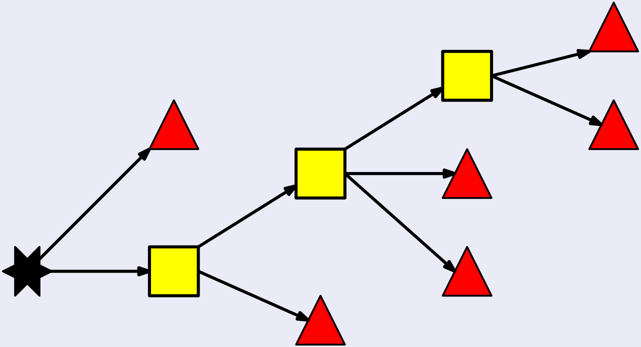
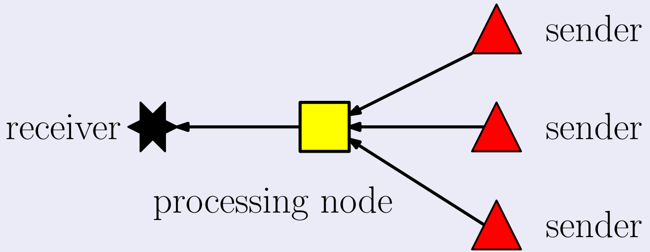


Figure: Hierarchy of processing nodes

Communication Schemes: Aggregation

processing = merge + reroute



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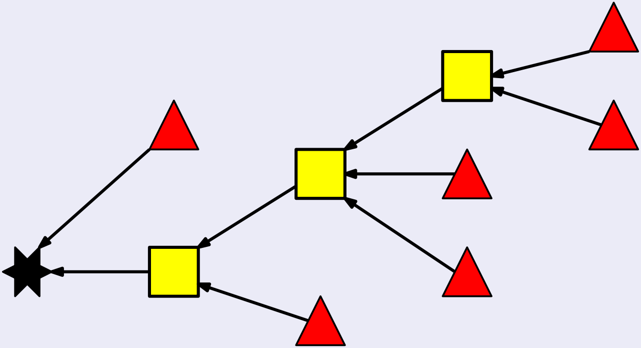


Figure: Hierarchy of processing nodes

Problem Statement

Enablers: Network Virtualization, e.g. SDN + NFV

- (Unsplittable) routes can be established arbitrarily
- Network functions can be placed on specific nodes

High-Level Questions

- How to compute *virtual* aggregation / multicasting trees?
  - Where to place in-network processing functionality?
  - How to trade-off between traffic and processing?

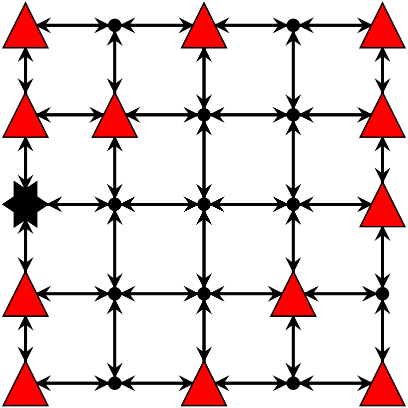
Introductory Example

Aggregation scenario

grid graph: 14 senders, one receiver

Virtualized links

data can be routed arbitrarily



Without in-network processing: Unicast

Solution Method

- minimal cost flow

Solution uses

- 41 edges
- 0 processing nodes

 receiver  sender

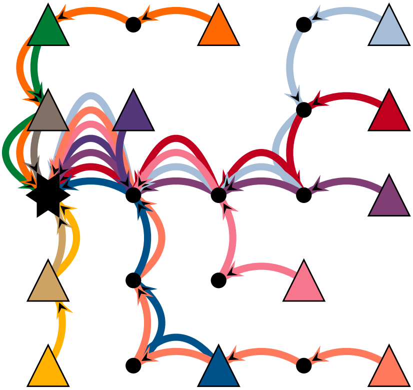


Figure: Unicast solution

With in-network processing at all nodes

Solution Method

- Steiner arborescence

Solution uses

- 16 edges
- 9 processing nodes

receiver	sender
processing node	sender with processing

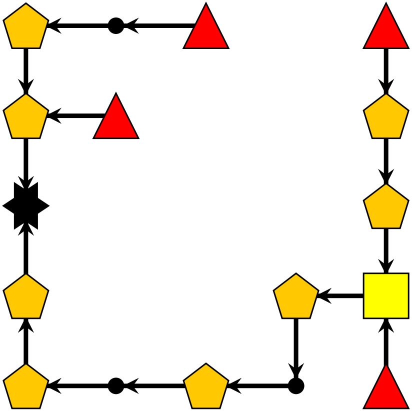
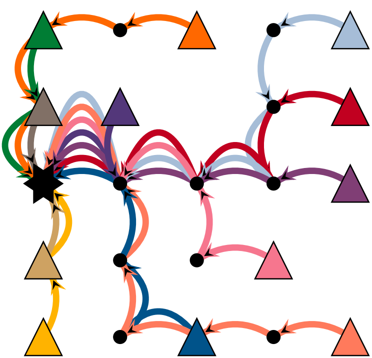
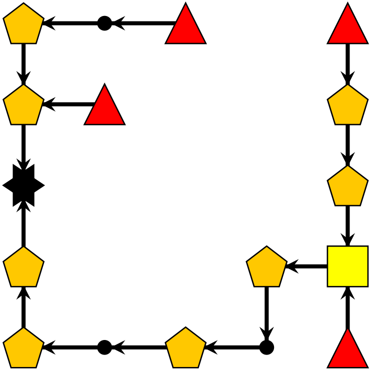


Figure: Aggregation tree

How to Trade-off?



vs.



What we aim for

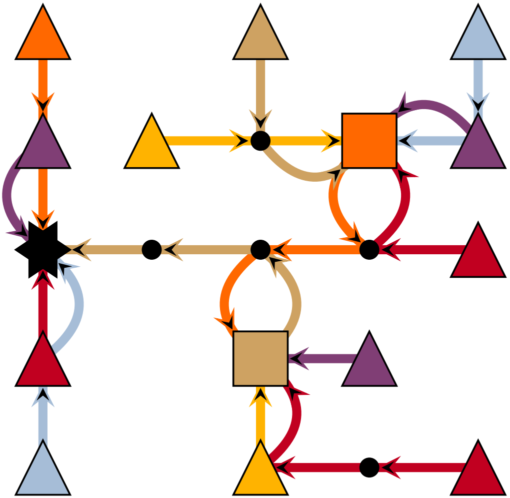
Solution uses

- 26 edges
- 2 processing nodes

receiver

sender

processing node



Solution Structure

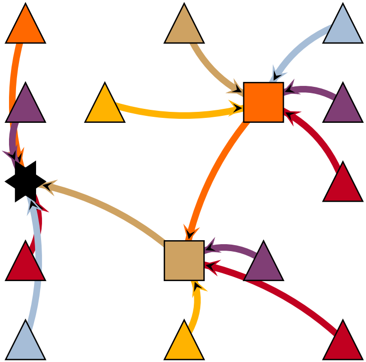


Figure: Virtual Arborescence

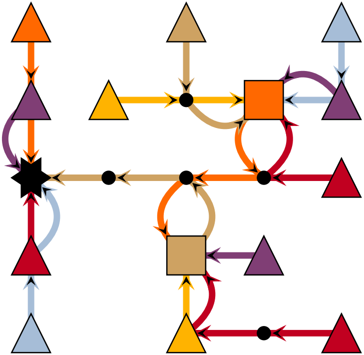


Figure: underlying routes

New Model: Constrained Virtual Steiner Arborescence Problem

Definition: CVSAP

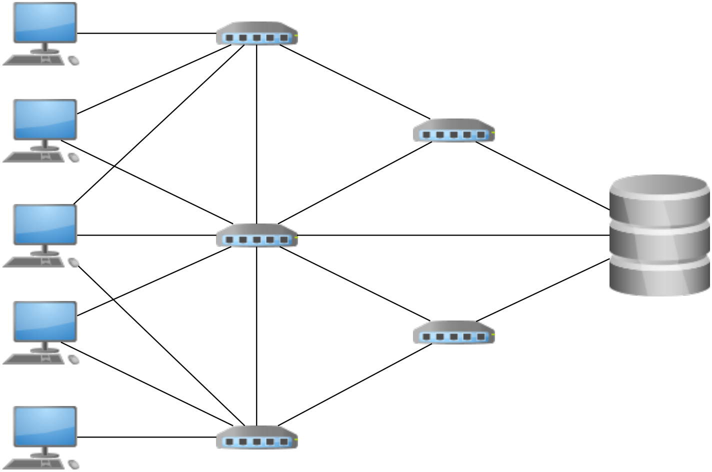
Find a Virtual Arborescence connecting senders to the single receiver, s.t.

1. bandwidth of substrate is not exceeded,
2. inner nodes are capable of processing flow,
3. the processing nodes' capacities are not exceeded,

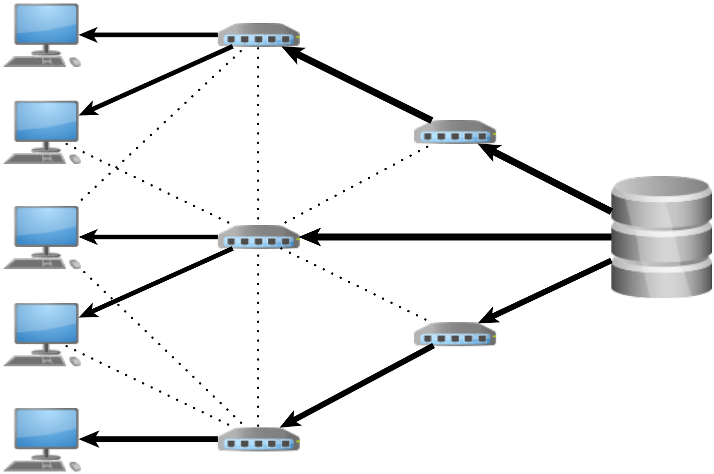
minimizing the joint cost for bandwidth allocations and function placement.

Applications

Service Replication

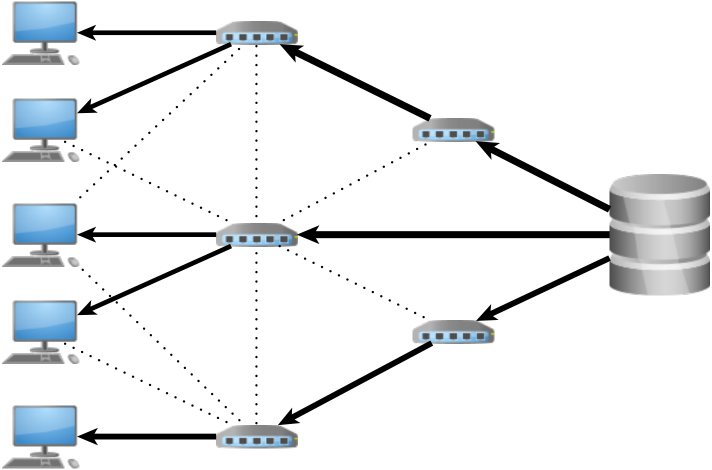


Service Replication

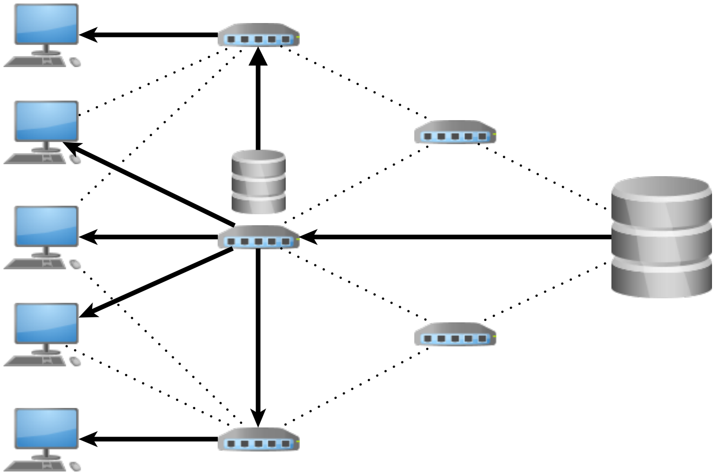


Service Replication

What if backend links are congested?

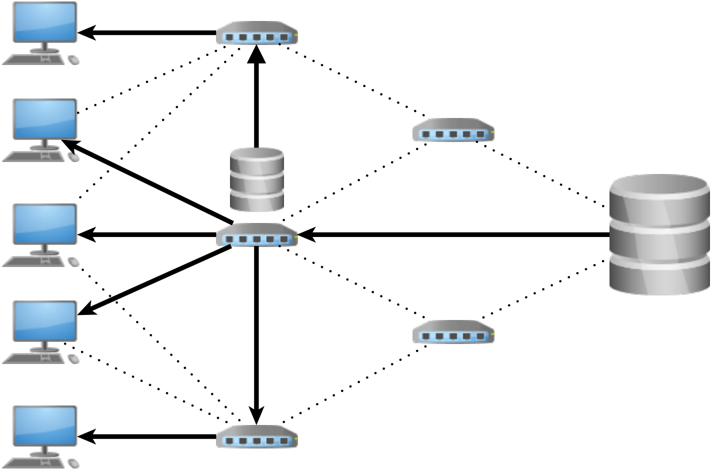


Service Replication

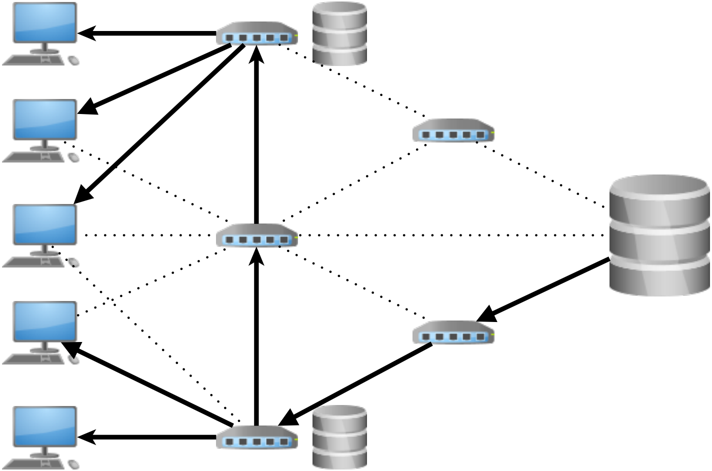


Service Replication

What if only '3' users can be handled?



Service Replication



Applications

	Network	Application	Technology, e.g.
multicast	ISP	service replication / cache placement [10, 11]	middleboxes / NFV + SDN
	backbone	optical multicast [6]	ROADM + SDH
	all	application-level multicast [15]	different
aggregation	sensor network	value & message aggregation [5, 8]	source routing
	ISP	network analytics: Gigascope [3]	middleboxes / NFV + SDN
	data center	big data / map-reduce: Camdoop [2]	SDN

edge capacities

processing node locations

processing node capacities

Applications: UNIFY / Network Analytics

EU FP7 IP UNIFY [4]

- Considers *service chaining* in the wide-area network, connecting e.g. customers at home to (possibly multiple) datacenter
- Business perspective: SLAs must be guaranteed strictly, otherwise fines
  - KPIs need to be monitored constantly
  - Different measurements need to be collected the whole time

unifying cloud
and carrier networks

Information Distribution

- Use multicast variant of CVSAP to distribute measurements.
- Placing processing nodes everywhere should be avoided due to the synchronization overhead (latencies).

Applications: UNIFY / Network Analytics

EU FP7 IP UNIFY [4]

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- Business perspective: SLAs must be guaranteed strictly, otherwise fines
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unify
unifying cloud
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Information Aggregation

- Use aggregation variant of CVSAP to compute (subfunctions) of the KPIs on-the-fly
- Processing nodes may offer multicast functionality (see above) as well.

Solution Approaches

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Wishful thinking: there exists a

- polynomial time algorithm
- solving CVSAP to optimality
- considering all constraints.

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Theorem: Inapproximability of CVSAP

Finding a feasible solution is NP-complete!

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- polynomial time algorithm
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Theorem: Inapproximability of CVSAP

Finding a feasible solution is NP-complete!

Approximations

- polynomial
- quality guarantee
- weaker models

Exact Algorithms

- non-polynomial
- optimality
- full model

Heuristics

- polynomial
- no solution guarantee
- full model

Comprehensive algorithmic study

Algorithms

Approximations

- NVSTP
- VSTP
- VSAP

Exact Algorithms

- multi-commodity flow
- single-commodity flow
  $\rightarrow$ VirtuCast

LP-based Heuristics

- FlowDecoRound
- MultipleShots
- GreedyDiving

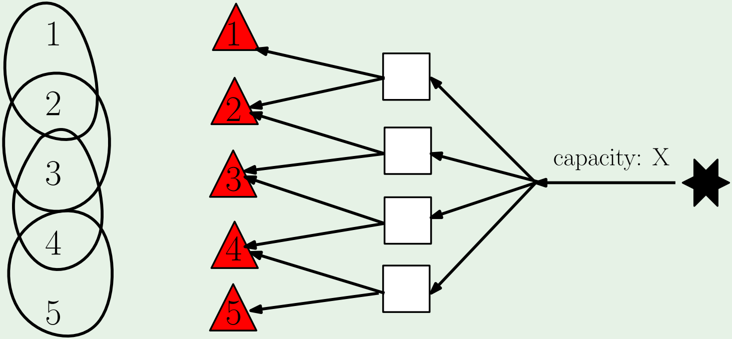
Combinatorial Heuristic

- GreedySelect

Inapproximability

Inapproximability

Reduction from Set Cover: Does a set cover of size X exist?



Theorem:

Finding *a feasible* solution is already NP-complete.

Approximation Algorithms for Variants

Variants

$$\begin{tikzcd} & \textbf{Directed} & \textbf{Undirected} \\ \textbf{edge and node capacities} & \mathrm{CVSAP} & \mathrm{CVSTP} \arrow[l] \\ \textbf{node capacities} & \mathrm{NVSAP} \arrow[u] & \mathrm{NVSTP} \arrow[l] \arrow[u] \\ \textbf{no capacities} & \mathrm{VSAP} \arrow[u] & \mathrm{VSTP} \arrow[l] \arrow[u] \end{tikzcd}$$

Approximation via related problems

Results

$$
\begin{tikzcd}
 & & \textbf{Directed} & \textbf{Undirected} & \\
\textbf{both capacities} & & \text{CVSAP} & \text{CVSTP} \arrow[l] & \\
\textbf{node capacities} & & \text{NVSAP} \arrow[u] & \text{NVSTP} \arrow[l] \arrow[u] \arrow[r, dashed, "\color{red}\mathcal{O}(\log,\log)"] & \text{DNSTP} \\
\textbf{no capacities} & \text{SAP} \arrow[r, leftrightarrow, "\color{red}\mathcal{O}(\log)"] & \text{VSAP} \arrow[u] & \text{VSTP} \arrow[l] \arrow[u] \arrow[r, dashed, "\color{red}\mathcal{O}(8)"] & \text{CFLP}
\end{tikzcd}
$$

Bottom Line

- Better understanding of how to incorporate *virtualized links*.
- Obtained lower bounds and approximations

Exact Algorithms for CVSAP

Overview

Why exact algorithms matter

- allow trading-off runtime with solution quality
- baseline for heuristics

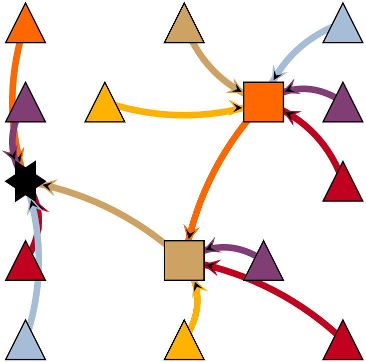
Choice: Integer Programming (IP)

- successfully employed for solving related problems (STP, CFLP, ...)
- generates lower bounds on-the-fly

Multi-Commodity Flow (MCF) Integer Program

First approach: MCF IP

- explicitly represent virtual arborescence
- necessitates independent construction of paths for all processing nodes



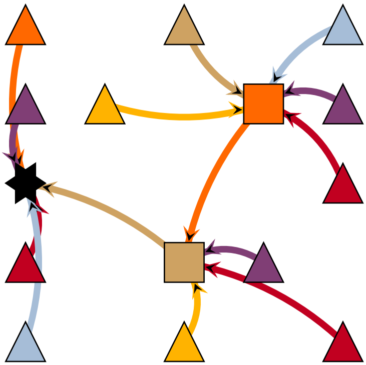
Multi-Commodity Flow (MCF) Integer Program

First approach: MCF IP

- explicitly represent virtual arborescence
- necessitates independent construction of paths for all processing nodes

Does not scale well

- number of binary variables: #processing nodes · #edges



Integer Program 1: A-CVSAP-MCF

$$
\begin{array}{llll}
\text{minimize} & C_{\text{MCF}} = \sum_{e \in E_G} \mathbf{c}_e (f_e + \sum_{s \in S} f_{s,e}) & & \text{(MCF-OBJ)} \\
& \quad + \sum_{s \in S} \mathbf{c}_s \cdot x_s & & \\
\text{subject to} & f^T(\delta^+_{E_{\text{MCF}}}(v)) = f^T(\delta^-_{E_{\text{MCF}}}(v)) + |\{v\} \cap T| & \forall\ v \in V_G & \text{(MCF-1)} \\
& f^s(\delta^+_{E^s_{\text{MCF}}}(v)) = f^s(\delta^-_{E^s_{\text{MCF}}}(v)) + \delta_{s,v} \cdot x_s & \forall\ s \in S, v \in V_G & \text{(MCF-2)} \\
& f^T_e + \sum_{s \in S} f^s_e \leq \begin{cases} \mathbf{u}_s x_s, & e = (s, \text{o}^-), s \in S \\ \mathbf{u}_r & , e = (r, \text{o}^-) \\ \mathbf{u}_e & , e \in E_G \end{cases} & \forall e \in E_{\text{MCF}} & \text{(MCF-3)} \\
& -|S|(1 - f^s_{\bar{s},\text{o}^-}) \leq p_s - p_{\bar{s}} - 1 & \forall\ s, \bar{s} \in S & \text{(MCF-4)} \\
& f^s_{(\bar{s},\text{o}^-)} \leq x_{\bar{s}} & \forall\ s \in S, \bar{s} \in S - s & \text{(MCF-5}^\star\text{)} \\
& f^s_{s,\text{o}^-} = 0 & \forall\ s \in S & \text{(MCF-6}^\star\text{)} \\
& f^s_{\bar{s},\text{o}^-} + f^{\bar{s}}_{s,\text{o}^-} \leq 1 & \forall\ s, \bar{s} \in S & \text{(MCF-7}^\star\text{)} \\
& x_s \in \{0, 1\} & \forall\ s \in S & \text{(MCF-8)} \\
& f^T_e \in \mathbb{Z}_{\geq 0} & \forall\ e \in E_{\text{MCF}} & \text{(MCF-9)} \\
& \color{red}{f^s_e \in \{0, 1\}} & \color{red}{\forall\ s \in S, e \in E_{\text{MCF}}} & \text{(MCF-10)} \\
& p \in [0, |S| - 1] & \forall\ s \in S & \text{(MCF-11)}
\end{array}
$$

Single-Commodity Flow IP

Single-commodity flow formulation

- computes *aggregated* flow on edges independently of the origin
- does not represent virtual arborescence

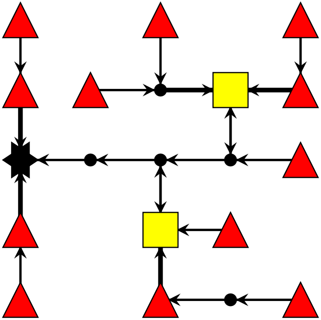


Figure: Single-commodity

Multi- vs Single-Commodity

Example: 6000 edges and 200 Steiner sites

- Single-commodity: 6000 integer variables
- Multi-commodity: 1,200,000 binary variables

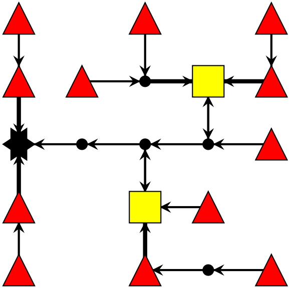


Figure: Single-commodity

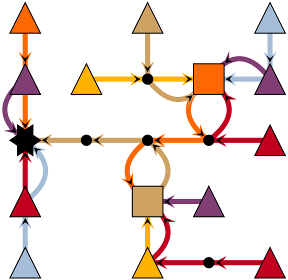


Figure: Multi-commodity

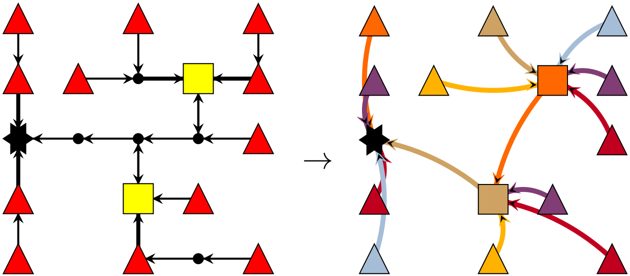
VirtuCast Algorithm

VirtuCast Algorithm

Outline of VirtuCast

1. Solve single-commodity flow IP formulation.
2. Decompose IP solution into Virtual Arborescence.

How to decompose?



(a) IP solution (b) Virtual Arborescence

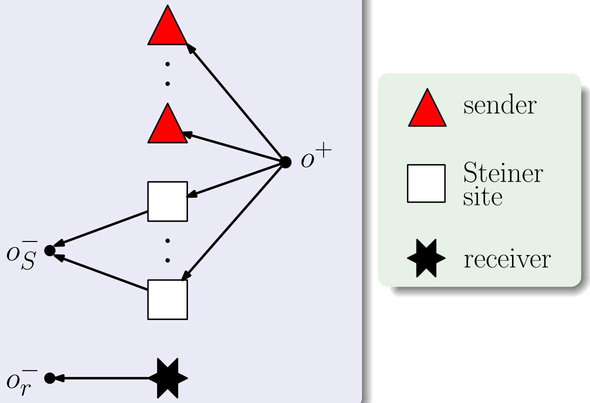
IP Formulation

Extended Graph

Additional nodes

- source o^+
- sinks o_r^- and o_S^-

Additional edges



IP Formulation I

$$
\begin{array}{lll}
\text{minimize} & C_{\text{IP}}(x,f) = \sum_{e \in E_G} \mathbf{c}_e f_e + \sum_{s \in S} \mathbf{c}_s x_s & \\
\text{subject to} & f(\delta^{+}_{E_{\text{ext}}}(v)) = f(\delta^{-}_{E_{\text{ext}}}(v)) & \forall\, v \in V_G \\
& f(\delta^{+}_{E^{R}_{\text{ext}}}(W)) \geq x_s & \forall\, W \subseteq V_G, s \in W \cap S \neq \emptyset \\
& f_e = 1 & \forall\, e = (\mathrm{o}^{+}, t) \in E^{T^{+}}_{\text{ext}} \\
& f_e = x_s & \forall\, e = (\mathrm{o}^{+}, s) \in E^{S^{+}}_{\text{ext}} \\
& x_s \in \{0,1\} & \forall\, s \in S \\
& f_e \in \mathbb{Z}_{\geq 0} & \forall\, e \in E_{\text{ext}}
\end{array}
$$

Connectivity Inequalities

STP Excursion [7]

$$
\text{(uSP)} \quad
\begin{array}{rll}
\min & c^T x & \\
(i) & x(\delta(W)) \geq 1, & \text{for all } W \subset V, W \cap T \neq \emptyset, \\
 & & (V \setminus W) \cap T \neq \emptyset, \\
(ii) & 0 \leq x_e \leq 1, & \text{for all } e \in E, \\
(iii) & x \text{ integer},
\end{array}
$$

Connectivity Inequalities

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(iii) & x \text{ integer}, &
\end{array}
$$

$\forall\ W \subseteq V_G, s \in W \cap S \neq \emptyset.\ \ f(\delta^+_{E^R_{\text{ext}}}(W)) \geq x_s$

'*From each activated Steiner site, there exists a path towards* o_r^-.'

Exponentially many constraints, but ...

can be separated in polynomial time.

Complete Formulation

$$
\begin{aligned}
\text{minimize} \quad & C_{\mathrm{IP}}(x,f) = \sum_{e \in E_G} \mathbf{c}_e f_e + \sum_{s \in S} \mathbf{c}_s x_s \\
\text{subject to} \quad & f(\delta^{+}_{E_{\mathrm{ext}}}(v)) = f(\delta^{-}_{E_{\mathrm{ext}}}(v)) && \forall\, v \in V_G \\
& f(\delta^{+}_{E^{R}_{\mathrm{ext}}}(W)) \geq x_s && \forall\, W \subseteq V_G, s \in W \cap S \neq \emptyset \\
& f_e \leq \mathbf{u}_s x_s && \forall\, e = (s, \mathrm{o}^{-}_{S}) \in E^{S^{-}}_{\mathrm{ext}} \\
& f_{(r,\mathrm{o}^{-}_{r})} \leq \mathbf{u}_r && \\
& f_e \leq \mathbf{u}_e && \forall\, e \in E_G \\
& f_e = 1 && \forall\, e \in E^{T^{+}}_{\mathrm{ext}} \\
& f_e = x_s && \forall\, e = (\mathrm{o}^{+}, s) \in E^{S^{+}}_{\mathrm{ext}} \\
& x_s \in \{0,1\} && \forall\, s \in S \\
& f_e \in \mathbb{Z}_{\geq 0} && \forall\, e \in E_{\mathrm{ext}}
\end{aligned}
$$

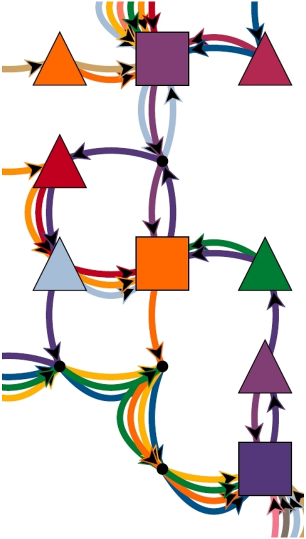
Decomposing flow is non-trivial (5 pages proof)!

Flow solution . . .

- contains cycles and
- represents *arbitrary* hierarchies.

Main Results

- decomposition is *always* feasible
- constructive proof:
  polynomial time algorithm



Outline of Decomposition Algorithm

Iterate

1. select a terminal t
2. construct path P from t towards o_r^- or o_S^-
3. remove one unit of flow along P
4. connect t to the second last node of P and remove t

After each iteration

Problem size reduced by one.

Outline of Decomposition Algorithm

Reduced problem must be feasible

Removing flow must not invalidate any connectivity inequalities.

Principle: Repair & Redirect

- decrease flow on path edge by edge
- if connectivity inequalities are violated

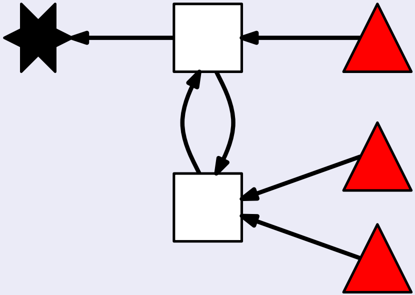
repair	increment flow on edge to remain feasible
redirect	choose another path from the current node

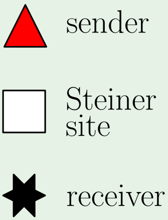
Theorem

Given an optimal solution, the Decomposition Algorithm computes a Virtual Arborescence in time $\mathcal{O}\left(|V_G|^2 \cdot |E_G| \cdot (|V_G| + |E_G|)\right)$.

Example

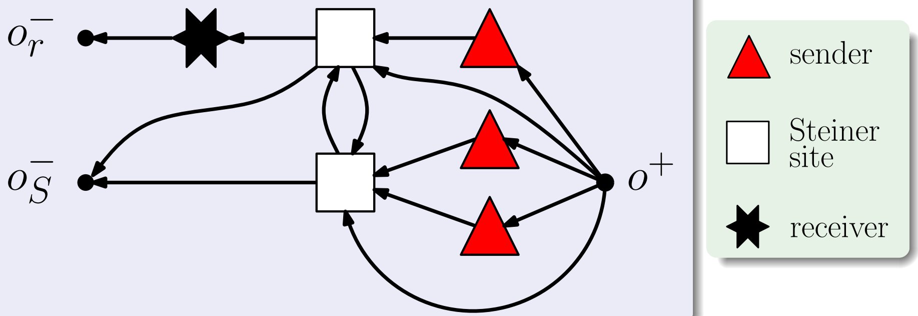
Scenario





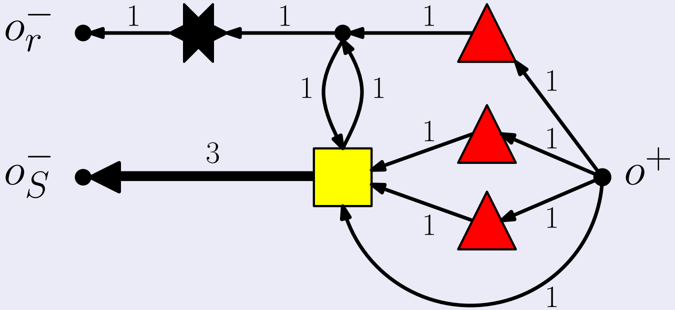
Example

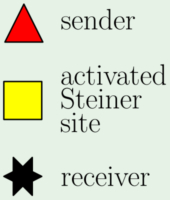
Extended Graph



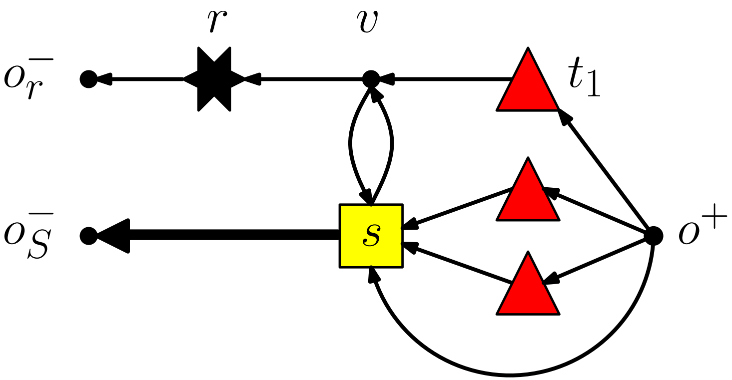
Example

Solution



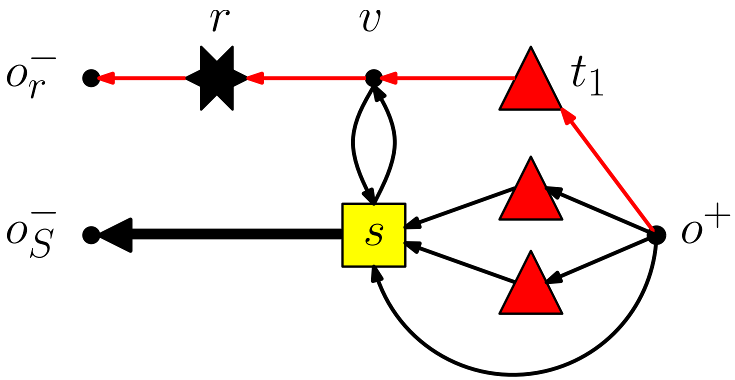


Decomposition Example I



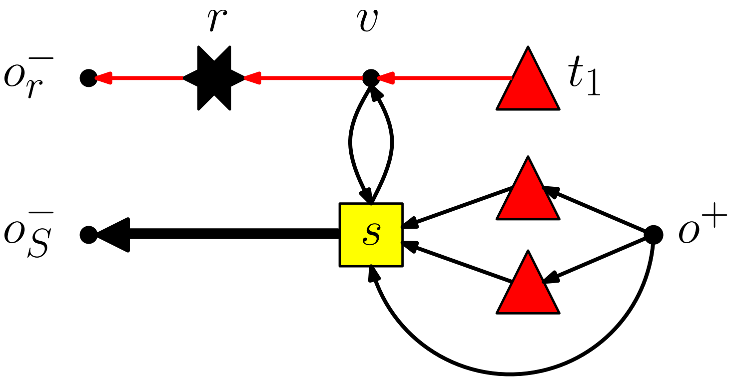
Decomposition Example I

$$P = \langle o^+, t_1, v, r, o_r^- \rangle$$



Decomposition Example I

$$P = \langle \mathsf{o}^+, t_1, v, r, \mathsf{o}_r^- \rangle$$



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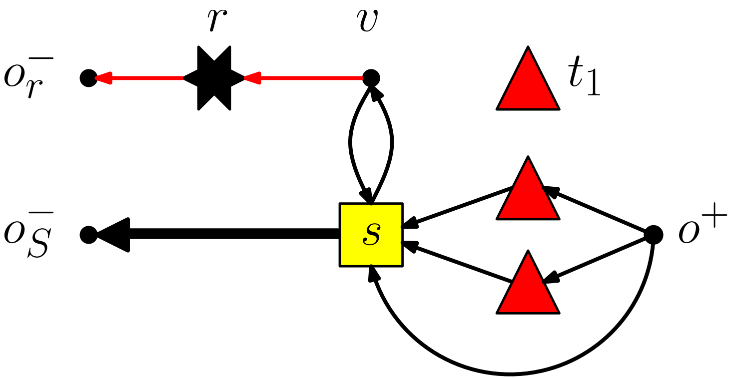

Decomposition Example I

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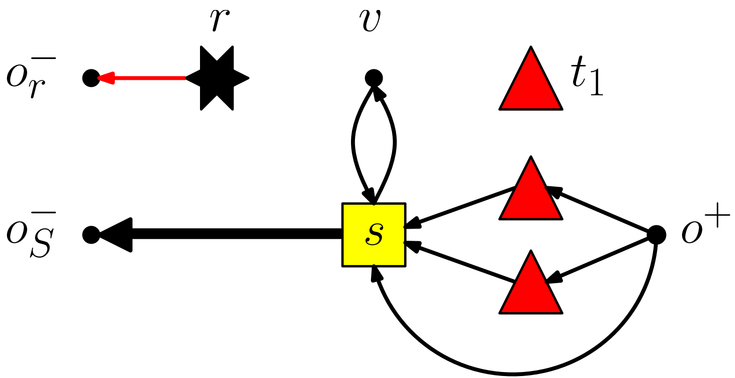
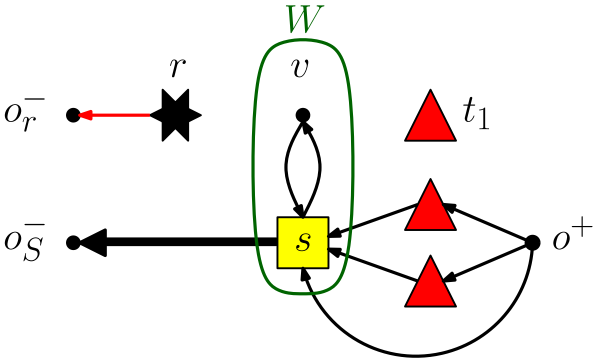


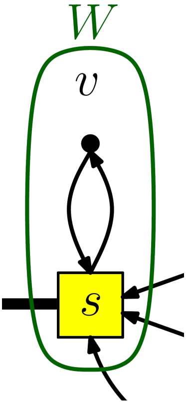
Redirecting Flow



Violation of Connectivity Inequality

$$f(\delta^+_{E^R_{\text{ext}}}(W)) \geq x_s \qquad \forall\ W \subseteq V_G, s \in W \cap S \neq \emptyset$$

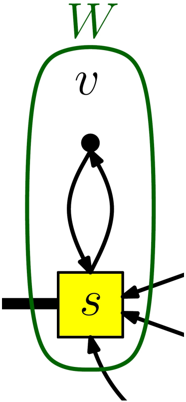
Redirecting Flow



Redirection towards o_S^- is possible!

There exists a path from v towards o_S^- in W.

Redirecting Flow



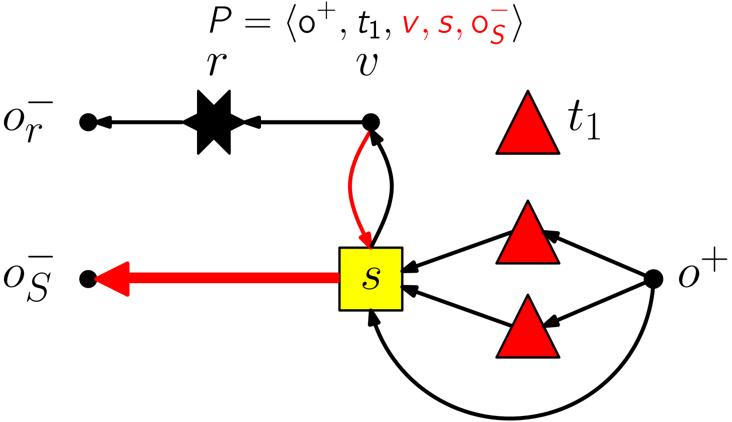
Redirection towards o_S^- is possible!

There exists a path from v towards o_S^- in W.

Reasoning

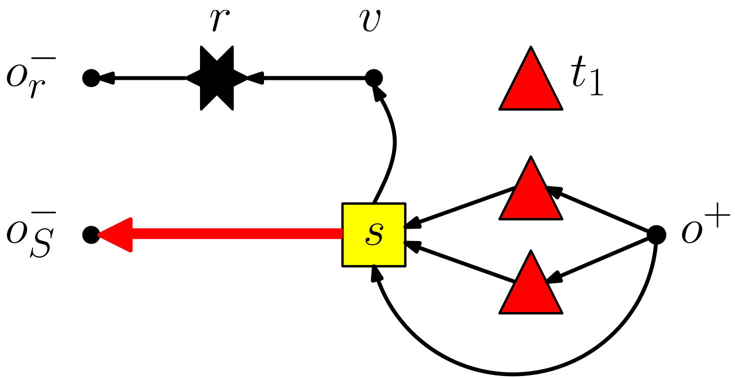
1. Flow preservation holds within W.
2. s could reach o_r^- via v before the reduction of flow.
3. v receives at least one unit of flow.
4. Flow leaving v must eventually terminate at o_S^-.

Decomposition Example II

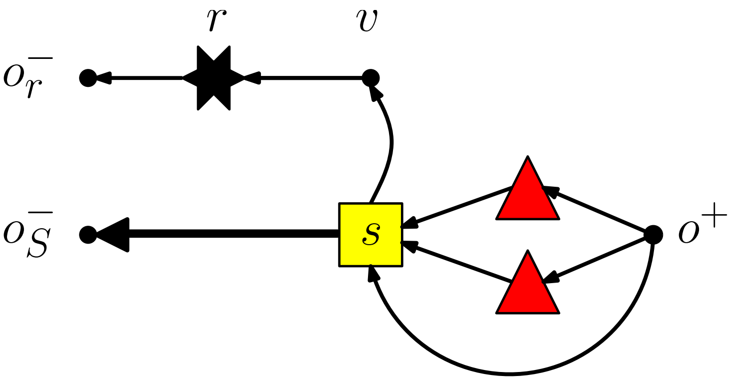


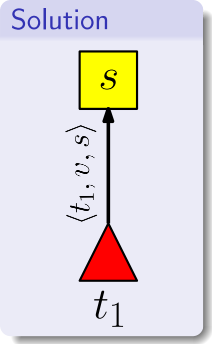
Decomposition Example II

$$P = \langle \mathsf{o}^+, t_1, v, s, \mathsf{o}_S^- \rangle$$

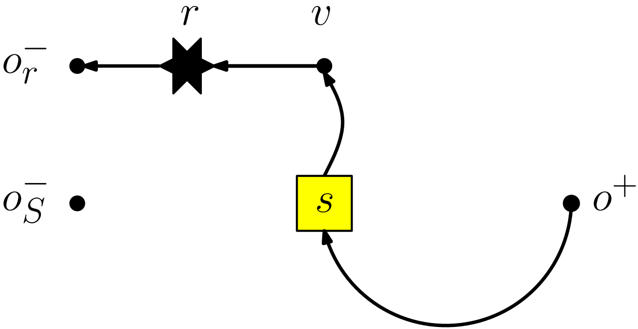


Decomposition Example II

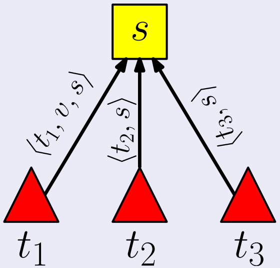




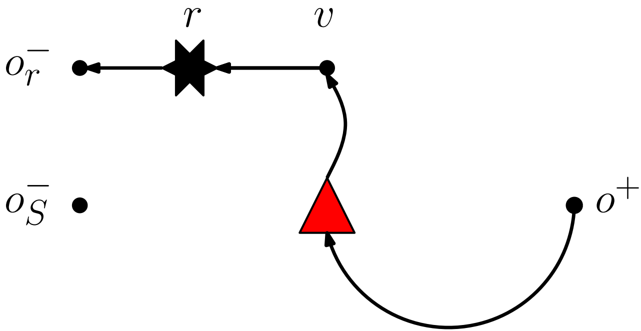
Decomposition Example II



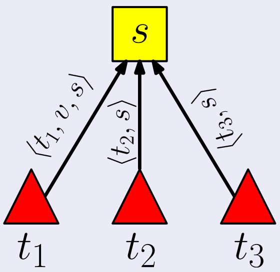
Solution



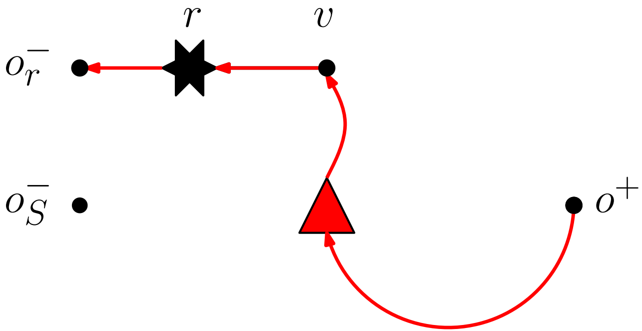
Decomposition Example II



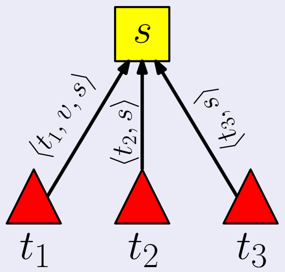
Solution



Decomposition Example II

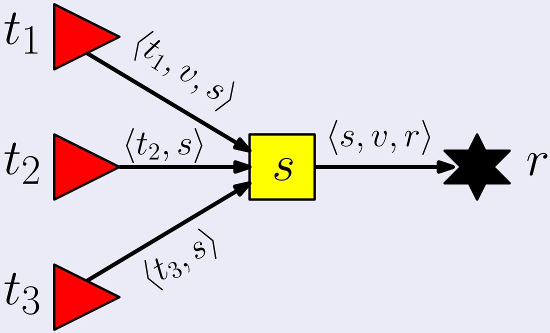


Solution



Decomposition Example II

Final Solution



Combinatorial Heuristic: GreedySelect

Combinatorial Heuristics

On Chickens and Eggs

- How and when to place processing nodes?
- How and when to reserve bandwidth for routes?
- How to react to infeasibilities?

Our Approach

- Place processing functionality and reserve bandwidth jointly.
- Try to avoid infeasibilities by proactive routing decisions.

GreedySelect Heuristic

Greedily either ...

- connect a single node to the connected component of the receiver or
- connect multiple nodes to an inactive processing node

minimizing the averaged discounted cost per connected node.

Selecting processing node + terminals + paths : $\mathcal{O}(|V| \cdot |E| + |V|^2 \log |V|)$

compute $\mathcal{P}_{\bar{s}} \triangleq (\bar{s} \in \bar{S}, T' \subseteq \bar{T}, \mathcal{P}_{T'} = \{P_{t,\bar{s}} | t \in T'\})$,

such that $P_{t,\bar{s}}$ connects t to $\bar{s}$,

$u^{\bar{s}}(e) - |\mathcal{P}_{T'}[e]| \geq 0$ for all $e \in E_G$,

$2 \leq |T'| \leq u_{r,S}(\bar{s})$

minimizing $c_{\bar{s},T'} \triangleq \left(\sum_{t \in T'} (c_E(P_{t,\bar{s}}) - c_E(P_{t,R})) + c_E(P_{\bar{s},R}) + c_S(\bar{s}) \right) / |T'|$

LP-based Heuristics

Overview

Linear Relaxations

- The linear relaxation of an IP is obtained by relaxing the integrality constraints of the variables, thereby obtaining a Linear Program (LP).
- Solutions to linear relaxations are readily availabe when using branch-and-bound to solve an IP.
- May provide useful information to guide the construction of a solution.

Usage

- LP-based heuristics are employed within the VirtuCast *solver* to improve the bounding process.
- Yield polynomial time heuristics when used together with the root relaxation.

FlowDecoRound Heuristic

- computes a *flow* decomposition and connects nodes randomly according to the decomposition
- processing nodes are activated if another node node connects to it, must be connected itself
- failsafe: shortest paths

Algorithm 1: FlowDecoRound

Input : Network $G = (V_G, E_G, c_E, u_E)$, Request $R_G = (r, S, T, u_r, c_S, u_S)$, LP relaxation solution $(\hat{x}, \hat{f}) \in \mathcal{F}_{\text{LP}}$ to **??**

Output: A Feasible Virtual Arborescence $\hat{\mathcal{T}}_G$ or `null`

set $\hat{S} \triangleq \emptyset$ **and** $\hat{T} \triangleq \emptyset$ **and** $U = T$
set $\hat{V}_{\mathcal{T}} \triangleq \{r\},\ \hat{E}_{\mathcal{T}} \triangleq \emptyset$ **and** $\hat{\pi} : \hat{E}_{\mathcal{T}} \to \mathcal{P}_G$
set $u(e) \triangleq \begin{cases} u_E(e) & \text{, if } e \in E_G \\ u_r(r) & \text{, if } e = (r, \text{o}_r^-) \\ u_S(s) & \text{, if } e = (s, \text{o}_S^-) \in E_{\text{ext}}^{S-} \\ 1 & \text{, else} \end{cases}$ **for all** $e \in E_{\text{ext}}$
while $U \neq \emptyset$ **do**
 choose $t \in U$ uniformly at random **and set** $U \leftarrow U - t$
 set $\Gamma_t \triangleq \texttt{MinCostFlow}\left(G_{\text{ext}}, \hat{f}, \hat{f}(\text{o}^+, t), t, \{\text{o}_S^-, \text{o}_r^-\}\right)$
 set $\hat{f} \leftarrow \hat{f} - \sum_{(P,f) \in \Gamma_t, e \in P} f$
 set $\Gamma_t \leftarrow \Gamma_t \setminus \{(P, f) \in \Gamma_t | \exists e \in P. u(e) = 0\}$
 set $\Gamma_t \leftarrow \Gamma_t \setminus \{(P, f) \in \Gamma_t | (\hat{V}_{\mathcal{T}} + t, \hat{E}_{\mathcal{T}} + (t, P_{|P|-1}))$ is not acyclic $\}$
 if $\Gamma_t \neq \emptyset$ **then**
  choose $(P, f) \in \Gamma_t$ with probability $f / \left(\sum_{(P_j, f_j) \in \Gamma_t} f_j\right)$
  if $P_{|P|-1} \notin \hat{V}_{\mathcal{T}}$ **then**
   set $U \leftarrow U + P_{|P|-1}$ **and** $\hat{V}_{\mathcal{T}} \leftarrow \hat{V}_{\mathcal{T}} + P_{|P|-1}$
  set $\hat{V}_{\mathcal{T}} \leftarrow \hat{V}_{\mathcal{T}} + t$ **and** $\hat{E}_{\mathcal{T}} \leftarrow \hat{E}_{\mathcal{T}} + (t, P_{|P|-1})$ **and** $\hat{\pi}(t, P_{|P|-1}) \triangleq P$
  set $u(e) \leftarrow u(e) - 1$ **for all** $e \in P$
set $u(e) \leftarrow 0$ **for all** $e = (s, \text{o}_S^-) \in E_{\text{ext}}^{S-}$ **with** $s \in S \wedge s \notin \hat{V}_{\mathcal{T}}$
set $\bar{T} \triangleq (T \setminus \hat{V}_{\mathcal{T}}) \cup (\{s \in S \cap \hat{V}_{\mathcal{T}} | \delta^+_{\hat{E}_{\mathcal{T}}}(s) = 0\})$
for $t \in \bar{T}$ **do**
 choose $P \leftarrow \texttt{ShortestPath}\left(G_{\text{ext}}^u, c_E, t, \{\text{o}_S^-, \text{o}_r^-\}\right)$ **such that** $(\hat{V}_{\mathcal{T}} + t, \hat{E}_{\mathcal{T}} + (t, P_{|P|-1}))$ is acyclic
 if $P = \emptyset$ **then**
  return `null`
 set $\hat{V}_{\mathcal{T}} \leftarrow \hat{V}_{\mathcal{T}} + t$ **and** $\hat{E}_{\mathcal{T}} \leftarrow \hat{E}_{\mathcal{T}} + (t, P_{|P|-1})$ **and** $\hat{\pi}(t, P_{|P|-1}) \triangleq P$
 set $u(e) \leftarrow u(e) - 1$ **for all** $e \in P$
for $e \in \hat{E}_{\mathcal{T}}$ **do**
 set $P \triangleq \hat{\pi}(e)$
 set $\hat{\pi}(e) \leftarrow \langle P_1, \ldots, P_{|P|-1} \rangle$
set $\hat{\mathcal{T}}_G \triangleq$ Virtual Arborescence $(\hat{V}_{\mathcal{T}}, \hat{E}_{\mathcal{T}}, r, \hat{\pi})$
return $\texttt{PruneSteinerNodes}(\hat{\mathcal{T}}_G)$

Intermezzo: VCPrimConnect

Important Observation

If all placed processing nodes are already connected, all senders can be assigned *optimally* using a minimum cost flow.

Outline

1. connect all opened processing nodes in tree via a adaption of Prim's MST algorithm
2. assign all sending nodes using min-cost flow

Algorithm 2: VCPrimConnect

Input : Network $G = (V_G, E_G, c_E, u_E)$, Request $R_G = (r, S, T, u_r, c_S, u_S)$, *Partial* Virtual Arborescence $\mathcal{T}_G^P = (V_\mathcal{T}^P, E_\mathcal{T}^P, r, \pi^P)$

Output: Feasible Virtual Arborescence $\mathcal{T}_G = (V_\mathcal{T}, E_\mathcal{T}, r, \pi)$ or `null`

- **set** $U \triangleq \{v | v \in V_\mathcal{T}^P \setminus \{r\}, \delta^+_{E_\mathcal{T}^P}(v) = 0\}$
- **set** $\bar{S} \triangleq U \cap S$
- **set** $V_\mathcal{T} \triangleq V_\mathcal{T}^P$, $E_\mathcal{T} \triangleq E_\mathcal{T}^P$ **and** $\pi(u,v) = \pi^P(u,v)$ **for all** $(u,v) \in E_\mathcal{T}$
- **set** $u(e) \triangleq u_E(e) - |\pi(E_\mathcal{T})[e]|$ **for all** $e \in E_G$
- **while** $\bar{S} \neq \emptyset$ **do**
  - **compute** $R \leftarrow \{r' | r \in \{r\} \cup (V_\mathcal{T} \cap S), r'$ reaches r in $\mathcal{T}_G, \delta^-_{E_\mathcal{T}}(r') < u_{r,S}(r')\}$
  - **compute** $P = \texttt{MinAllShortestPath}(G^u, c_E, \bar{S}, R)$
  - **if** $P = \mathtt{null}$ **then**
    - **return** $\mathtt{null}$
  - **end**
  - **set** $\bar{S} \leftarrow \bar{S} - P_1$
  - **set** $E_\mathcal{T} \leftarrow E_\mathcal{T} + (P_1, P_{|P|})$ **and** $\pi(P_1, P_{|P|}) \triangleq P$
  - **set** $u(e) \leftarrow u(e) - 1$ **for all** $e \in P$
- **end**
- **set** $\bar{T} \triangleq U \cap T$
- **set** $u_V(r') \triangleq u_{r,S}(r') - \delta^-_{E_\mathcal{T}}(r')$ **for all** $r' \in \{r\} \cup (V_\mathcal{T} \cap S)$
- **compute** $\Gamma = \{P^{\bar{t}}\} \leftarrow \texttt{MinCostAssignment}(G, c_E, u, u_V, \bar{T}, \{r\} \cup V_\mathcal{T} \cap S)$
- **if** $\Gamma = \emptyset$ **then**
  - **return** $\mathtt{null}$
- **end**
- **set** $E_\mathcal{T} \leftarrow E_\mathcal{T} + (t, P^t_{|P^t|})$ **and** $\pi(t, P^t_{|P^t|}) \triangleq P^t$ **for all** $P^t \in \Gamma$
- **return** $\mathcal{T}_G \triangleq (V_\mathcal{T}, E_\mathcal{T}, r, \pi)$

MultipleShots

- treats node variables as probabilities and iteratively places processing functionality accordingly
- tries to generate a feasible solution in each round via VCPrimConnect

Algorithm 3: MultipleShots

Input : Network $G = (V_G, E_G, c_E, u_E)$, Request $R_G = (r, S, T, u_r, c_S, u_S)$, LP relaxation solution $(\hat{x}, \hat{f}) \in \mathcal{F}_{\text{LP}}$ to ??

Output: A Feasible Virtual Arborescence $\hat{\mathcal{T}}_G$ or `null`

set $\lfloor S \rfloor \triangleq \{s \in S | \hat{x}_s \leq 0.01\}$ **and** $\lceil S \rceil \triangleq \{s \in S | \hat{x}_s \geq 0.99\}$
`addConstraintsLocally`$(\{x_s = 0 | s \in \lfloor S \rfloor\} \cup \{x_s = 1 | s \in \lceil S \rceil\})$
set $\dot{S}_0 \triangleq \lfloor S \rfloor \cup$ **and** $\dot{S}_1 \triangleq \lceil S \rceil$
`disableGlobalPrimalBound()`
repeat
    $(\hat{x}, \hat{f}) \leftarrow$ `solveSeparateSolve()`
    if `infeasibleLP()` **return** *null*
    set $\lfloor S \rfloor \triangleq \{s \in S | \hat{x}_s \leq 0.01\}$ **and** $\lceil S \rceil \triangleq \{s \in S | \hat{x}_s \geq 0.99\}$
    `addConstraintsLocally`$(\{x_s = 0 | s \in \lfloor S \rfloor\} \cup \{x_s = 1 | s \in \lceil S \rceil\})$
    set $\dot{S}_0 \leftarrow \dot{S}_0 \cup \lfloor S \rfloor$ **and** $\dot{S}_1 \leftarrow \dot{S}_1 \cup \lceil S \rceil$
    set $\hat{S} \triangleq S \setminus (\dot{S}_0 \cup \dot{S}_1)$
    if $\hat{S} \neq \emptyset$ **then**
        repeat
            set $S_1 \triangleq \hat{S}$
            remove s **from** S_1 **with probability** $1 - \hat{x}_s$ **for all** $s \in S_1$
            if $S_1 = \emptyset$ ***and*** $|S \setminus (\dot{S}_0 \cup \dot{S}_1)| < 10$ **then**
                set $S_1 \leftarrow S \setminus (\dot{S}_0 \cup \dot{S}_1)$
        until $S_1 \neq \emptyset$
        `addConstraintsLocally`$(\{x_s = 1 | s \in S_1\})$
        set $\dot{S}_1 \leftarrow \dot{S}_1 \cup S_1$
    $\hat{\mathcal{T}}^P_G \triangleq (\hat{V}^P_T, \hat{E}^P_T, r, \emptyset)$ **where** $\hat{V}^P_T \triangleq \{r\} \cup T \cup \dot{S}_1$ **and** $\hat{E}_T \triangleq \emptyset$
    set $\hat{\mathcal{T}}_G \triangleq$`VCPrimConnect`$(G, R_G, \hat{\mathcal{T}}^P_G)$
    if $\hat{\mathcal{T}}_G \neq$ *null* **then**
        return `PruneSteinerNodes`$(\hat{\mathcal{T}}_G)$
until $\dot{S}_0 \cup \dot{S}_1 = S$
return *null*

GreedyDiving

- aims at generating a feasible *IP* solution
- iteratively bounds at least a single variable from below, first fixing node variables
- complex failsafe: PartialDecompose + VCPrimConnect

Algorithm 4: GreedyDiving

Input : Network $G = (V_G, E_G, c_E, u_E)$, Request $R_G = (r, S, T, u_r, c_S, u_S)$, LP relaxation solution $(\hat{x}, \hat{f}) \in \mathcal{F}_{\mathrm{LP}}$ to ??

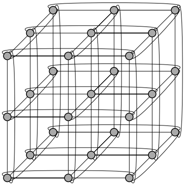
Output: A Feasible Virtual Arborescence $\hat{\mathcal{T}}_G$ or `null`

set $\lfloor S \rfloor \triangleq \{s \in S | \hat{x}_s \leq 0.01\}$ **and** $\lceil S \rceil \triangleq \{s \in S | \hat{x}_s \geq 0.99\}$
`addConstraintsLocally`$(\{x_s = 0 | s \in \lfloor S \rfloor\} \cup \{x_s = 1 | s \in \lceil S \rceil\})$
set $\dot{S} \triangleq \lfloor S \rfloor \cup \lceil S \rceil$ **and** $\dot{E} \triangleq \emptyset$
do
- $(\hat{x}', \hat{f}') \leftarrow$ `solveSeparateSolve()`
- **if** *infeasibleLP()* ***and*** $\dot{S} = S$ **then**
  - **break**
- **else if** *infeasibleLP()* ***or*** *objectiveLimit()* **then**
  - **return** *null*
- **set** $(\hat{x}, \hat{f}) \leftarrow (\hat{x}', \hat{f}')$
- **if** $\dot{S} \neq S$ **then**
  - **set** $\lfloor S \rfloor \triangleq \{s \in S | \hat{x}_s \leq 0.01\}$ **and** $\lceil S \rceil \triangleq \{s \in S | \hat{x}_s \geq 0.99\}$
  - `addConstraintsLocally`$(\{x_s = 0 | s \in \lfloor S \rfloor\} \cup \{x_s = 1 | s \in \lceil S \rceil\})$
  - **set** $\dot{S} \leftarrow \dot{S} \cup \lfloor S \rfloor \cup \lceil S \rceil$
  - **set** $\hat{S} \triangleq S \setminus \dot{S}$
  - **if** $\hat{S} \neq \emptyset$ **then**
    - **choose** $\hat{s} \in \hat{S}$ **with** $c_S(\hat{s}) / \hat{x}_{\hat{s}}$ **minimal**
    - `addConstraintsLocally`$(\{x_{\hat{s}} = 1\})$
    - **set** $\dot{S} \leftarrow \dot{S} + \hat{s}$
- **else if** $\dot{E} \neq E_{\mathrm{ext}}$ **then**
  - **set** $\lfloor E \rfloor \triangleq \{e \in E_{\mathrm{ext}} | \, |\hat{f}_e - \lfloor \hat{f}_e \rfloor| \leq 0.001\}$, $\lceil E \rceil \triangleq \{e \in E_{\mathrm{ext}} | \, |\hat{f}_e - \lceil \hat{f}_e \rceil| \leq 0.001\}$
  - `addConstraintsLocally`$(\{f_e = \lfloor \hat{f}_e \rfloor | e \in \lfloor E \rfloor\} \cup \{f_e = \lceil \hat{f}_e \rceil | e \in \lceil E \rceil\}$
  - **set** $\dot{E} \leftarrow \dot{E} \cup \lfloor E \rfloor \cup \lceil E \rceil$
  - **set** $\hat{E} \triangleq E_{\mathrm{ext}} \setminus \dot{E}$
  - **if** $\hat{E} \neq \emptyset$ **then**
    - **choose** $\hat{e} \in \hat{E}$ **with** $\lceil \hat{f}_{\hat{e}} \rceil - \hat{f}_{\hat{e}}$ **minimal**
    - `addConstraintsLocally`$(\{\hat{f}_{\hat{e}} \geq \lceil \hat{f}_{\hat{e}} \rceil\})$
    - **set** $\dot{E} \leftarrow \dot{E} + \hat{e}$
- **else**
  - **break**

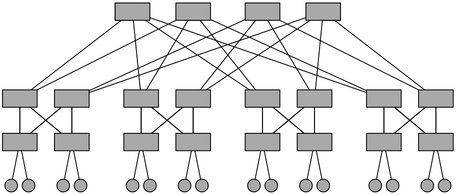
set $\hat{f}_e \leftarrow \lfloor \hat{f}_e \rfloor$ **for all** $e \in E_{\mathrm{ext}} \setminus \dot{E}$
set $\hat{\mathcal{T}}_G^P \leftarrow$ `PartialDecompose` $(G, R_G, (\hat{x}, \hat{f}))$
return `VCPrimConnect`$(G, R_G, \hat{\mathcal{T}}_G^P)$

Computational Evaluation

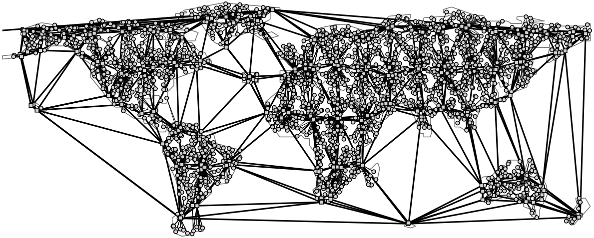
Topologies



3D torus



Fat tree



An ISP topology generated by IGen with 2400 nodes.

Instances

Generation Parameters

- five graph sizes I-V
- 15 instances per graph size: different Steiner costs, different edge capacities

	Nodes	Edges	Processing Locations	Senders
Fat tree	1584	14680	720	864
3D torus	1728	10368	432	864
IGen	4000	16924	401	800

Table: Largest graph sizes

Computational Setup

Implementation

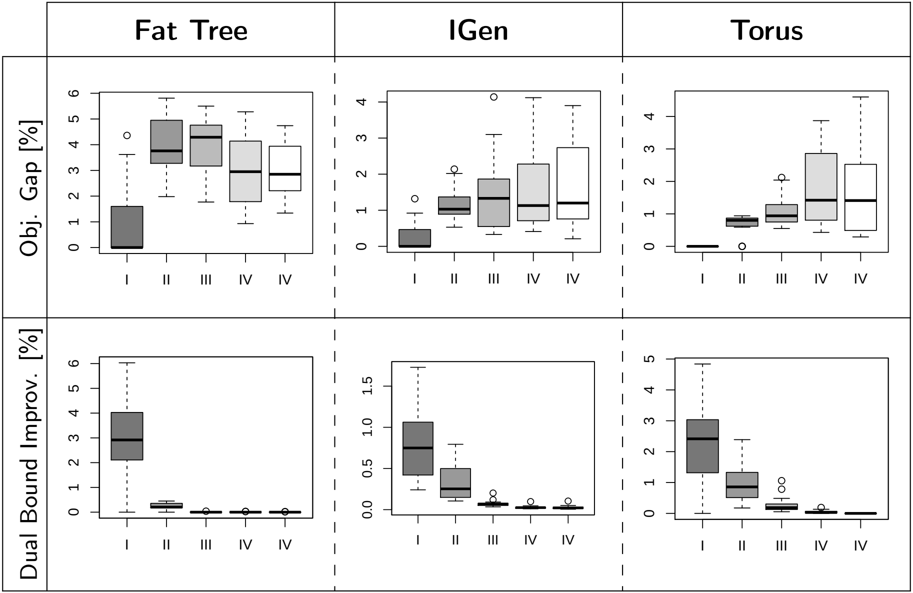
- *all* algorithms (except MCF-IP) are implemented in C/C++
- VirtuCast uses SCIP [1], many different parameters to consider
  - separation
  - branching
  - heuristics
  - separation procedure: nested cuts, creep flow, cyclic generation...
- MCF-IP is implemented using GMPL + CPLEX

Objective

Solve instances within reasonable time: 1 hour runtime limit

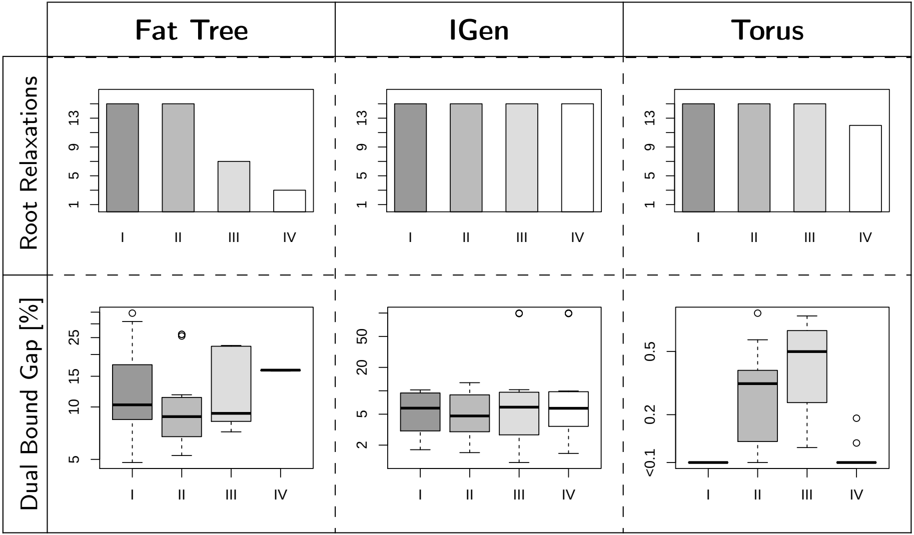
VirtuCast + LP-based Heuristics

VirtuCast + LP-based Heuristics



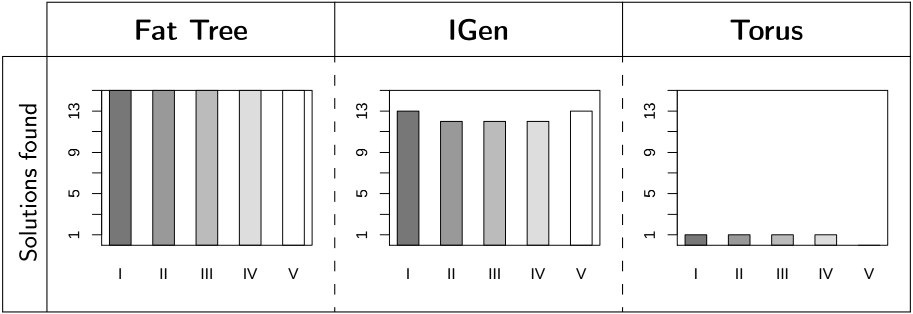
MCF-IP

MCF-IP: Performance

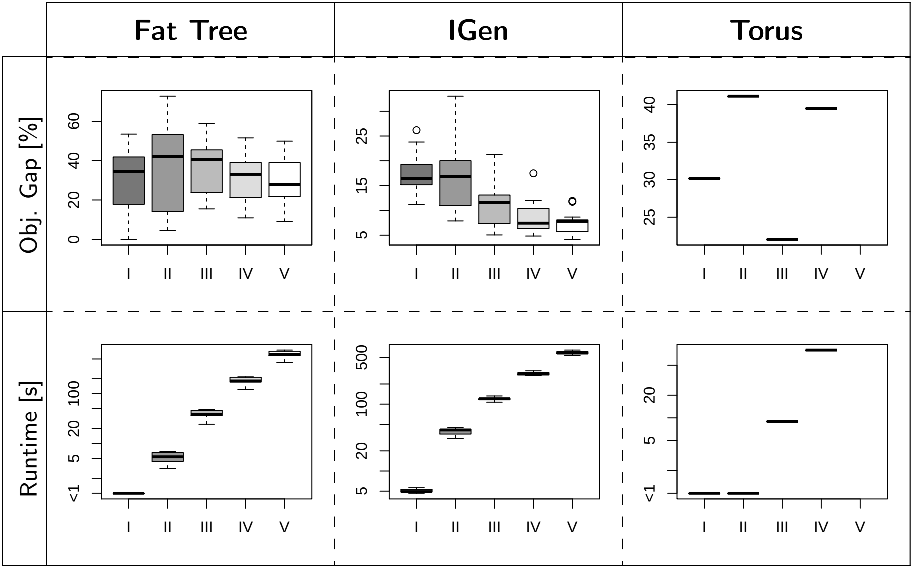


GreedySelect

GreedySelect: Efficacy

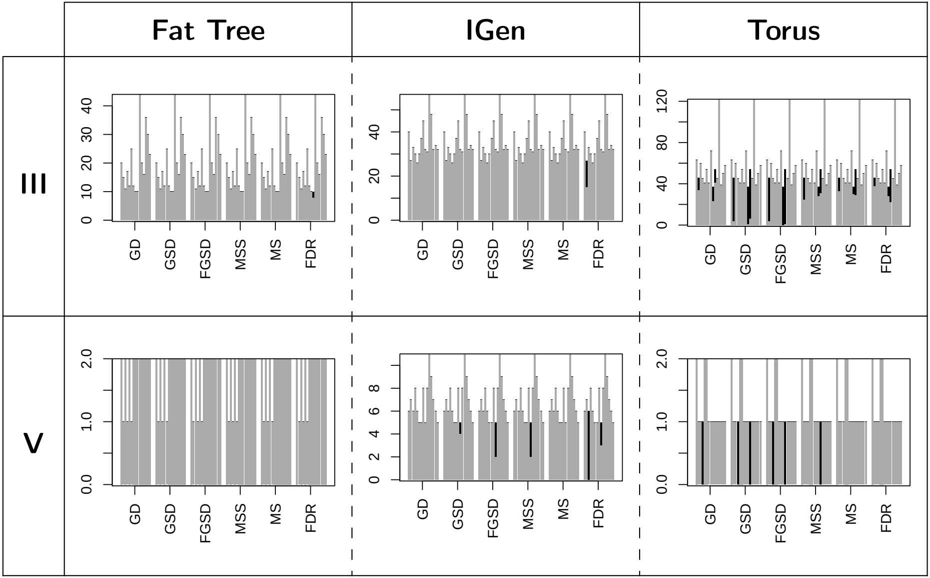


GreedySelect: Performance

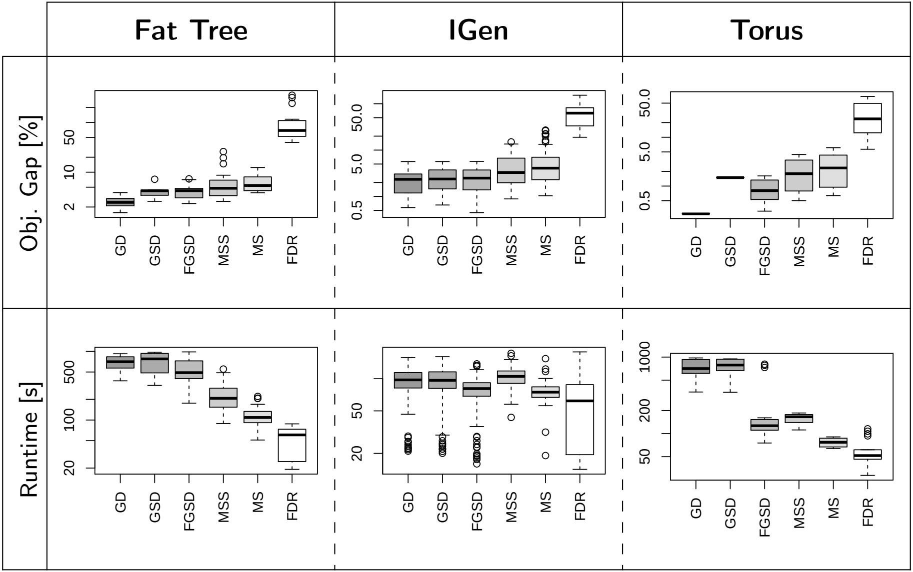


LP-based Heuristics

LP-based Heuristics: Efficacy



LP-based Heuristics: Performance on graph size V



Conclusion

Publications

Matthias Rost, Stefan Schmid: OPODIS 2013 & arXiv [14, 13]
Matthias Rost (Adv. Stefan Schmid): M.Sc. Thesis [12]

Applications → Concise definition of CVSAP

Inapproximability

Approximations

- NVSTP
- VSTP
- VSAP

Exact Algorithms

- multi-commodity flow
- single-commodity flow → VirtuCast

Heuristics

- FlowDecoRound
- MultipleShots
- GreedyDiving
- GreedySelect

Extensive explorative Computational Evaluation

Related Work

Molnar: Constrained Spanning Tree Problems [9]

- Shows that optimal solution is a 'spanning hierarchy' and not a DAG.

Oliveira et. al: Flow Streaming Cache Placement Problem [11]

- Consider a weaker variant of multicasting CVSAP without bandwidth
- Give weak approximation algorithm

Shi: Scalability in Overlay Multicasting [15]

- Provided heuristic and showed improvement in scalability.

Future Work

Model Extensions

- prize-collecting variants
- concurrent multicast / aggregation sessions

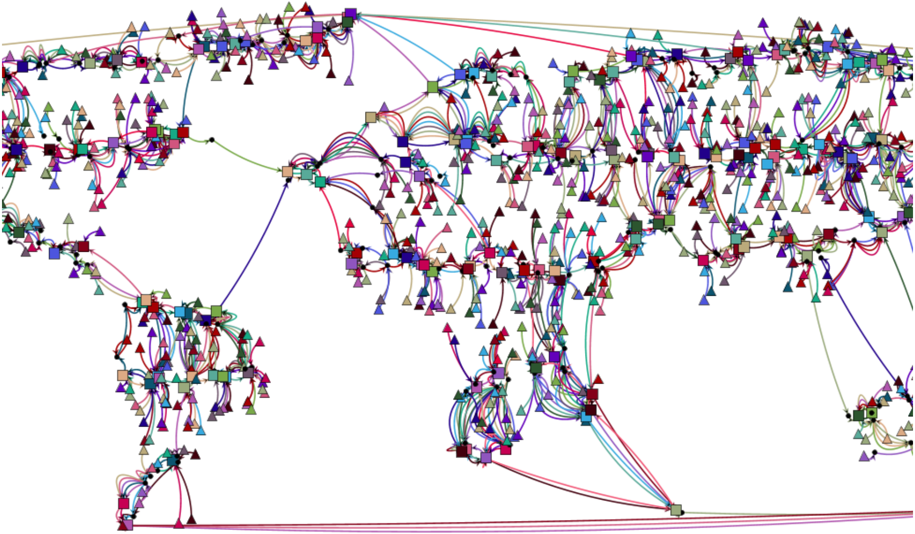
Application Modeling

- Stratosphere II: Big Data
- UNIFY Project: flow analytics

IP formulation

- try to derive further cuts / facets

Thanks



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